QP Code : 77031

(2 ½ Hours) **[Total Marks : 75**

N.B. : (1) **All** questions are **compulsory**
(2) **Figures to the right indicate maximum marks.**
(3) Use of non-programmable Calculators and Log-tables is allowed.
(4) Draw neat diagrams wherever necessary.

1. (A) Attempt **any one** : **10**

(i) (a) Solve $\frac{d^2y}{dx^2}+2\frac{dy}{dx}+y=x$ and

(b) Solve $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+y=2\cos x$

(ii) Discuss the method of solving the second order homogeneous linear ordinary differential equations with the constant coefficients. **10**

(B) Attempt **any one**

(i) Test the following equation for exactness and find its solution **5**

$$(2x^3-3x^2y+y^3)\frac{dy}{dx}=2x^3-6x^2y+3xy^2$$

(ii) The equation of motion of a damped simple harmonic oscillator is given by - **5**

$$\frac{d^2y}{dt^2}+2b\frac{dy}{dt}+\omega^2y=0$$

Find its solution

2. (A) Attempt **any one** : **10**

(i) Expand the following function in Fourier series.

$f(x) = 0$ for $-\pi \leq x < 0$

$= h$ for $0 \leq x \leq \pi$

Hence show that

$$f(x)=\frac{h}{2}+\frac{2h}{\pi}\sum_{\substack{n=1\\ \text{odd}}}^{\infty}\frac{\sin nx}{n}$$

Graphically represent f (x) in the internal $[-\pi, \pi]$ and outside.

[TURN OVER]

(ii) (a) State the complex form of Fourier series and hence describe formal development of Fourier transform pair. **10**

(b) Find Fourier transform of f(x) given by

$$f(x)=\begin{cases} x & -1\le x\le 1 \\ 0 & |x|>1 \end{cases}$$

(B) Attempt **any one** :

(i) Show the change of interval of Fourier series from $[-\pi, \pi]$ to $[-\ell, \ell]$. Extend the interval to involve all values of x and obtain the Fourier integral formula. **5**

(ii) Find sine transform of **5**

$f(x) = e^{-x}$

3. (A) Attempt **any one** :

(i) Show that, for an infinitesimal general interaction, $TdS = dU + PdV$. **10**

(ii) Consider a system in contact with a heat reservoir at temperature T. Find the probability that the system is in quantum state r of energy U_r **10**

Define partition function z and show that

$$\overline{U}=\frac{\sum U_r\, e^{-\beta U_r}}{z}$$

(B) Attempt **any one** :

(i) Describe Gibb's free energy (G) of a system. Discuss its variation during thermal interaction and at the equilibrium state. **5**

(ii) Discuss the concept of phase space. Hence find number of quantum states lying between p and p+dp. Here p is the momentum. **5**

4. (A) Attempt **any one** :

(i) What are fermions? Derive Fermi-Dirac distribution law. Hence discuss the Fermi energy. **10**

(ii) State and explain Boltzmann distribution law of energies. Use the law to obtain equations for root mean square speed, most probable speed and average seed of the molecules. **10**

(B) Attempt **any one** :

(i) Show that the average energy of a quantized oscillator is 5

$$E = \frac{h\nu}{e^{h\nu/kT} - 1}$$

(ii) Describe the concept of a priori probability and thermodynamic probability in relation to distribution of N balls in k cells. Hence write equation for the total probability. 5

5. (A) Attempt **any one** :

(i) The equation of motion of a body falling under gravity in a resistive medium is $\frac{dv}{dt} + bv = g$ 4

Solve this equation for v if the body starts from rest.

(ii) Solve $\frac{dN}{dt} = -\lambda N(t)$ by method of separation of variables. 4

(B) Attempt **any one** :

(i) Get the Fourier transforms of first order and second order derivatives of a function f (x). 4

(ii) State Dirichlet's theorem and explain Dirichlet's conditions. 4

(C) Attempt **any one** :

(i) Relative probability of the two states of particle in a system is e^3 at temperature 410 °C. Calculate energy difference between the two states. 4

(ii) Consider a system of six spin half particles fixed in uniform magnetic field B. Each particle has magnetic moment μ_o associated with it. Find various possible macrostates for the system and number of microstate in each macrostate. Which macrostate is most probable? Why? 4

(D) Attempt **any one** :

(i) Calculate number of modes of vibrations per unit volume in a black body cavity for the wavelengths between 6000 AU and 6010 AU. 3

(ii) Find number of possible arrangements of 6 particles in 8 energy cells assuming that they obey : (i) B-E statistics (ii) F-D statistics 3
