[Max Marks:75] Revised Course Duration: 21/2 Hours

N.B. 1. All questions are compulsory.

2. From Question 1,2 and 3, Attempt any one from part(a) and any two from part(b).

3. From Question 4, Attempt any THREE

4. Figures to the right indicate marks for the respective parts.

1 a i Define triple integral of a bounded function $f\colon Q \to \mathbb{R}$ where Q = $[a_1,b_1] \times [a_2, b_2] \times [a_3, b_3]$ is a rectangular box in $\mathbb{R}^3$. Further show with usual notations $m(b_1-a_1)(b_2-a_2)(b_3-a_3) \leq \iiint_Q f \leq M(b_1-a_1)(b_2-a_2)(b_3-a_3)$ 8

ii State and prove Fubini's theorem for a rectangular domain in $\mathbb{R}^2$

b i State the change of variables formula for triple integral, stating clearly the condition under which it is valid. Use it to express the triple integral $\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{0}^{\sqrt{4-x^2-y^2}} f(x,y,z)\, dzdydx$ in spherical co-ordinates $(\rho,\theta,\emptyset)$. 12

ii Evaluate $\int_0^1\int_{\sqrt{x}}^{3} e^{y^3}\, dydx$ by reversing the order of integration. Sketch the region of integration.

iii Evaluate $\iiint_S \; y\, dV$ where S is the solid enclosed by the planes z = 0, z = y and the parabolic cylinder $y = 1 - x^2$

iv Evaluate $\iint_R \frac{y-4x}{y+4x}\, dA$ where R is the region bounded by the lines $y = 4x, y = 4x+2, y = 2-4x, y = 5-4x$.

2 a i Let f be a continuously differentiable scalar field defined on an open set U in $\mathbb{R}^n$. Suppose C be a closed curve in U, with parameterization $r(t), t \in [a,b]$. then prove that $\oint_C \nabla f.\overline{dr} = 0$ 8

ii State and prove Green's Theorem for a rectangle. Further find the area of the region between two concentric circles of radii r_1 and r_2 where $r_1 < r_2$, using Green's theorem.

b i $F = (P,Q)$ is a continuously differentiable function defined on a simply connected region D in $\mathbb{R}^2$. Show that $\oint_C Pdx + Qdy = 0$ around every closed curve C in D if and only if $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}\ \forall\,(x,y) \in D$ 12

ii Evaluate $\int_C F$, where $F(x,y,z) = (x,^2 - xy\,, 1)$ and C is the circle of radius 1, with centre at the origin and lying in the yz, plane, traversed counterclockwise as viewed from the positive x axis.

iii Show that the line integral $\int_{(-1,2)}^{(3,1)} (y^2+2xy)dx + (x^2+2xy)\, dy$ is path independent. Further evaluate the line integral.

iv Use Green's theorem to find the area of the region D = $\left\{(x,y) \in \mathbb{R}^2 \ : x^{\frac{2}{3}} + y^{\frac{2}{3}} \leq 4\right\}$

3 a i State and prove Stoke's Theorem for an oriented smooth, simple parameterized surface in $\mathbb{R}^3$ bounded by a simple, closed curve traversed counter clockwise assuming general form of Green's Theorem. 8

	ii	For the surface $\bar{r}(u,v)$ described by the vector equation $\bar{r}(u,v) = X(u,v)i + Y(u,v)j + Z(u,v)\hat{k},\ (u,v) \in T$ where X, Y, Z are differentiable on T, define the fundamental vector product $\frac{\partial \bar{r}}{\partial u} \times \frac{\partial \bar{r}}{\partial v}$. If C is a smooth curve lying on the surface, $C = \bar{r}(\propto(t)),\ \propto: [a,b] \to T$, then show that $\frac{\partial \bar{r}}{\partial u} \times \frac{\partial \bar{r}}{\partial v}$ is normal to C at each point. Further assume S and C satisfies the hypotheses of Stokes' Theorem and f, g have continuous second order partial derivatives. Prove with usual notations that $\int_C (f \nabla f).\ d\bar{r} = 0$.	
b	i	Assuming S and V satisfy the conditions of the Divergence Theorem and scalar fields f and g, components of $\bar{F}$ have continuous partial derivatives, $\hat{n}$ is unit outward normal. Prove p) $\lvert V \rvert = \frac{1}{3}\iint_S \bar{r}.\ \hat{n} dS$ where $\bar{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ and $\lvert V \rvert$ = volume of V. q) $\iint_S curlF\ .\hat{n} dS = 0$.	12
	ii	Using Stoke's theorem evaluate $\iint_S curl\,\bar{F}.\hat{n}\,dS$ where $F(x,y,z) = x\,\hat{\imath} + z^2\,\hat{\jmath} + y^2\,\hat{k}$ and S is the plane surface $x + y + z = 1$ lying in the first octant.	
	iii	Evaluate $\iint_S yds$, where S is cylinder $x^2 + y^2 = 1, 0 \le z \le 1$	
	iv	Using Gauss Divergence Theorem to evaluate $\iint_S F \cdot ndS$ where $F(x,y,z) = (y - x, z - y, y - x)$ and S is the cube bounded by the planes $x = \pm 1, y = \pm 1, z = \pm 1$.	
4	i	Using Spherical coordinates find the volume of the solid region bounded by the surface $\rho = cos\varphi$.	15
	ii	Find the area of the region R bounded by the curves $y = \sin x$ and $y = \cos x$ for $0 \le x \le \frac{\pi}{4}$.	
	iii	A particle starts at the point (-2, 0) moves along the X-axis to (2, 0) and then along the semi circle $y = \sqrt{4 - x^2}$ to the starting point. use Green's theorem to find the work done on this particle by the force field F(x, y) = x $\hat{\imath}$ + $(x^3 + 3xy^2)\,\hat{\jmath}$	
	iv	Evaluate the line integral of $f(x,y,z) = e^{\sqrt{z}}$, along the path parametrised by $\gamma(t) = (1, 2, t^2), 0 \le t \le 1$	
	v	Find the surface area of S which is parametrically given by $r(\theta, z) = (a\cos\theta, a\sin\theta, z)\ and\ (\theta, z) \in [0, 2\pi] \times [-1, 1]$, $a > 0$ is constant.	
	vi	Using Stoke's theorem evaluate $\oint_C \bar{F}.\overline{dr}$ where $\bar{F}(x,y,z) = (x^3 + y^3)\,\hat{\imath} + (x - y)\,\hat{\jmath}$, C is the boundary of the rectangular lamina in the xy-plane. Bounded by the lines $x = 0, x = 2, y = 2\ and\ y = 5$ oriented counter clockwise as viewed from above.	