TY BSc Sem V Maths

Graph Theory and combinatories

Year - 2016-17 **Q.P. Code : 78963**

(OLD COURSE)

Duration:[2½Hours] [Total Marks: 75]

N.B. 1) All questions are compulsory.
2) Figures to the right indcate full marks.

1. (a) Attempt any ONE question: (8)
i. If $(A^n) = (a_{ij}^n)$ is the n^{th} power of adjacency matrix A of a graph G with $V(G) = \{v_1, v_2, \ldots v_n\}$, then prove that
(1) a_{ij}^2, $i \neq j$ is the number of $v_i - v_j$ path of length 2.
(2) $a_{ii}^2 = deg(v_i)$
(3) $\frac{1}{6}$trace of A^3 is the number of triangles in G.
ii. Show that a nontrivial graph is bipartite if and only if it contains no odd cycle.

(b) Attempt any TWO questions: (12)
i. Define cut edge of a graph G. Prove that an edge e of a graph G is a cut edge of G if and only if e is acyclic and hence prove that every edge in a tree is a cut edge.
ii. If $\delta(G)$ is minimum degree of G with $\delta(G) \geq (n-1)/2$, then show that G is connected. Is the bound $(n-1)/2$ sharp? In this case, can $(n-1)/2$ be replaced by$(n-2)/2$?
iii. Show that a connected (p, q) graph contains a cycle if and only if $q \geq p$.
iv. Show that every nontrivial graph contains at least two vertices which are non cut vertices.

2. (a) Attempt any ONE question: (8)
i. Let G be (p, q) graph. Show that the following statements are equivalent.
1) G is tree.
2) G is acyclic and $q = p - 1$.
3) G is connected and $q = p - 1$.
ii. For any simple graph G, prove that $\kappa(G) \leq \kappa'(G) \leq \delta(G)$ where $\kappa(G)$ denote the vertex connectivity and $\kappa'(G)$ denotes the edge connectivity and $\delta(G)$ denotes the minimum degree of a graph G.

(b) Attempt any TWO questions: (12)
i. Show that each label spanning tree with n vertices corresponds to a unique vector $s = (s_1, s_2, \ldots s_{n-2})$ where $s_i \in \{1, 2, ..., n\}$ for $i = 1, 2, ..., n$
ii. Let $\tau(G)$ denote the number of spanning trees of a graph G. If $e \in E(G)$ is not a loop, then prove that $\tau(G) = \tau(G-e) + \tau(G.e)$.
iii. Show that a tree with at least two vertices contains at least two pendant vertices.
iv. Show that there exist a connected graph with degree sequence $d_1 \geq d_2 \geq \ldots.. \geq d_n$ if and only if 1) $d_i \geq 1$, for $1 \leq i \leq n$ and 2) $\sum_{i=1}^{n} d_i = 2n - 2$

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3. (a) Attempt any ONE question:
   i. If u and v are non-adjacent vertices in a graph G such that $deg(u) + deg(v) \geq |V(G)|$. Show that G is hamiltonian if and only if $G + uv$ is hamiltonian.
   ii. Define cube graph Q_k. Show that cube graph Q_k, with $k \geq 2$ is a Hamiltonian graph.

   (b) Attempt any TWO questions:
   i. Does the dodecohedron have a hamiltonian cycle? Does it satisfy Dirac's Theorem? Jusify.
   ii. If G is a graph on p vertices with $p \geq 3$ such that $deg(u) + deg(v) \geq p - 1$ for every pair of non adjacent vertices u and v in G, then show that G contains a Hamiltonian path.
   iii. Let G be a simple graph with p vertices and q edges with $p \geq 3$. If $q \geq \frac{p^2-3p+6}{2}$ then prove that G is Hamiltonian.
   iv. If G is Hamiltonian graph then for every nonempty proper subset S of $V(G)$, prove that $\omega(G - S) \leq |S|$. Give an example of a graph which satisfies the above condition but not Hamiltonian.

4. Attempt any THREE questions:
   (a) If G is a simple graph on at least six vertices, then prove that either $K_3 \subseteq G$ or $K_3 \subseteq G^c$
   (b) Let G be a connected graph with at least three vertices. If $e(u, v)$ is a cut-edge in G, then show that either u or v is a cut-vertex.
   (c) State Huffman's algorithm for prefix code.
   (d) Show that a vertex v in a tree T is a cut vertex of T if and only if $deg(v) > 1$.
   (e) Prove that the cube graph Q_k is bipartite $k-$regular graph with 2^k vertices.
   (f) Show that the graph representing legal moves of a knight on a 3x7 chess board is not Hamiltonian.
