TYBSc Sem V - Maths
Mathematics - Elective Graph Theory

Q.P. Code : 78975

(REVISED COURSE)
Duration:[2½Hours] [Total Marks: 75]

N.B. 1) All questions are compulsory.
2) Figures to the right indcate full marks.

1. (a) Attempt any ONE question: (8)
   i. Show that a nontrivial graph is bipartite if and only if it contains no odd cycle.
   ii. Define a self complementary graph. If G is self complementary graph of order p, show that G is connected and $p \equiv 0$ or $1 (\mod 4)$

(b) Attempt any TWO questions: (12)
   i. Define adjacency matrix of a graph G. If G is a graph with vertex set $V(G) = \{v_1, v_2, ..., v_n\}$ and adjacency matrix $A = [a_{ij}]$, then show that the entry $a_{ij}^{(k)}$ in i^{th} row and j^{th} column of A^k is the number of distinct $v_i - v_j$ walks of length k in G.
   ii. If G and H are isomorphic graphs , then show that the degree sequence of the vertices of G are the same as the degree sequence of the vertices of H.
   iii. If G is a graph of size q, then prove that $\sum_{v \in V(G)} deg v = 2q$. Hence prove that every graph has an even number of odd vertices.
   iv. Explain Dijkstra's Algorithm to find shortest path in a graph G.

2. (a) Attempt any ONE question: (8)
   i. State and prove Cayley's formula for spanning trees.
   ii. Define a cut vertex of a graph G. Show that every nontrivial graph contains at least two vertices which are not cut vertices.

(b) Attempt any TWO questions: (12)
   i. Define spanning tree of a graph G. Show that a graph is connected if and only if it has a spanning tree.
   ii. Show that if a connected (p, q) graph G, $p \geq 3$, contains a cut edge, then it must contains a cut vertex. Is the converse true? Justify.
   iii. Show that the there exist a tree with degree sequence $d_1 \geq d_2 \geq ..... \geq d_n$ if and only if 1) $d_i \geq 1$, for $1 \leq i \leq n$ and 2) $\sum_{i=1}^{n} d_i = 2n - 2$
   iv. Explain and write Depth First Search Algorithm for finding a spanning tree.

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3. (a) Attempt any ONE question:

   i. If G is a graph on p vertices with $p \geq 3$ such that $deg(u) + deg(v) \geq p$ for every pair of non adjacent vertices u and v in G, then prove that G is Hamiltonian.

   ii. Show that a nontrivial connected graph G is Eulerian if and only if every vertex of G has even degree.

   (b) Attempt any TWO questions:

   i. Define a line graph of a graph. Show that line graphs $L(K_3)$ and $L(K_{1,3})$ are isomorphic.

   ii. Show that Q_n is a regular graph on 2^n vertices and $n2^{n-1}$ edges

   iii. Show that a simple graph is Hamiltonian if and only if its closure is Hamiltonian.

   iv. Let G be a simple graph with p vertices and q edges with $p \geq 3$. If $q \geq \frac{p^2-3p+6}{2}$ then prove that G is Hamiltonian.

4. Attempt any THREE questions:

   (a) If G is simple graph on p vertices and $\delta(G) \geq \frac{p-1}{2}$, then prove that G is a connected graph where $\delta(G)$ denotes the minimum degree of G.

   (b) State Havel-Hakimi Theorem for degree sequence of a graph G. Using this theorem check whether the sequence 5, 4, 3, 3, 2, 2, 2, 1 is graphical and if so, draw the graph with this degree sequence.

   (c) Describe Kruskal's algorithm for finding minimum spanning tree in a connected weighted graph.

   (d) Show that a vertex v in a tree T is a cut vertex of T if and only if $deg(v) > 1$.

   (e) Show that there are n edge disjoint Hamiltonian cycles in K_{2n+1}. Also draw all edge disjoint Hamiltonian cycles in K_7.

   (f) Describe Fluery's Algorithm to find a closed Eulerian trail.