VCD 7/10/19 SYBSC (PM) MATHS- II SEM-IV(Choice Based) **Marks:100**

3 Hours

Instructions: 1) All questions are compulsory.

2) Figures to right Indicate full marks.

Q.1 Choose correct alternative in each of the following (2 marks each)

1) Which of the following set is group under indicated binary operation.

a) $(\mathbb{N},+)$ b) $(\mathbb{R},o)$ c) $(\mathbb{R}^*,+)$ d) $(\mathbb{R},+)$

2) Inverse of an element of a group G___________

a) Need not unique b) is unique c) may be two d) None of these

3) In a group G, the number of element at G such that $a^2 = a$ is ___________

a) 0 b) 1 c) 2 d) None of these

4) The group of symmetries of a regular n-gon (n>3) has___________

a) n elements of order 2 & n-1 elements of order.

b) n elements of order 2 if n is odd.

c) Exactly 2 elements of order n.

d) None of these.

5) The number of elements of order 2 in S_4 is___________

a) 8 b) 6 c) 9 d) None of these.

6) Let H be a subgroup of G. $a,b \in G$ if $aH \neq bH$ then ___________

a) $aH \cap bH = \varphi$ b) $aH \cap bH \neq \varphi$ c) $aH \subseteq bH$ d) None of these

7) The left cossets of H={1,1,1} in U (30) are ___________

a) 7+H,13+H,19+H b) 7+H,13+H,23+H

c) H,1+H,29+H d) None of these.

8) Order of U(n), n>2 is ___________

a) Even b) odd c) n-1 d) None of these.

9) Number of homomorphisms from $\mathbb{Z}_{12}$ to $\mathbb{Z}_{30}$ is___________

a) 6 b) 7 c) 8 d) None of these.

10) The group of symmetries of S^o ___________

a) a square is abelian b) an equilateral triangle is abelian

c) a rectangle is abelian d) None of these.

Q.2 a) Attempt any ONE question from the following (8 marks each)

1) Let G be a group then prove that

i) Identity elements of G is unique.

ii) Inverse of an element in G is unique.

iii) Cancellation laws holds in G.

iv) $(a^{-1})^{-1} = a \ \forall a \in G$

2) State and prove necessary and sufficient condition for a nonempty set to be a subgroup.

Q.2 b) Attempt any two question from the following (6 marks each)

1) $(\mathbb{Z}^{*}_{n}, \odot)$ is a group iff n is prime, prove it.

2) let H be a finite subset of a group (G,*) then prove that H is a subgroup of G iff

$a*b \in H \ \forall a, b \in H$.

3) Let G be a group, $a \in G$. if $o(a)=n$ then prove that $o(a^m)= \frac{n}{(m,n)}$ where (m,n)= gcd of m,n.

4) If G is group of even order then show that G has an element of order two.

Q.3 (a) Attempt any one questions from the following (8 marks each)

1) Let G be a finite cyclic group of order n generated by a then prove that a^m is also generator of G iff $(m,n)=1$.

2) Define a cyclic group and prove that every finite cyclic group of order n has unique subgroup of order d for each divisor d of n.

Q.3 (b) Attempt any two questions from the following (6 marks each)

1) Let G be an infinite cyclic group generated by a. show that every nontrivial subgroup of G is an infinite cyclic and prove or disprove following .

G={(1,1), (1,-1), (-1,1), (-1,-1)} is a group under operation

(a,b)(c,d)=(ac,bd) but not a cyclic group.

2) Prove that every subgroup of cyclic group is cyclic.

3) Let G be a finite group then prove that a is generator of G iff $o(a)=o(G)$.

4) Show that every subgroup of prime order P is cyclic, further show that it has (P-1) generators.

Q.4 (a) Attempt any one questions from the following (8 marks each).

1) State and prove Lagrange's theorem for finite group.

2) Define kernel of group homomorphisms, prove that if f: $G \longrightarrow G^1$ is group homomorphisms then show that ker$f$ is subgroup of G and show that f injective iff ker f={e}.

Q.4 (b) Attempt any two questions from the following (6 marks each).

1) Define an automorphism of group . let $a \in G$ show that fa: $G \longrightarrow G$ defined by $fa(x)=axa^{-1}$ $\forall x \in G$ is an automorphism.

2) State and prove fermat's Little theorem.

3) Let H be a subgroup of group G,a. $b \in G$ then prove that

i) $a \in aH$ ii) $aH=bH$ or $aH \cap bH= \varphi$ iii) $aH=Ha$ iff $H=aHa^{-1}$

4) In a finite group, show that the order of each element of the group decides the order of the group.

Q.5 Attempt any four questions from the following. (5 marks each).

1) List all generators, all subgroups of cyclic group $(\mathbb{Z}_{15},+)$.

2) Show that the group U(8) is not isomorphic to U(10) but U(8) is isomorphic to U(12).

3) i) Find all subgroups of Klein's four group.

ii) Prove that every proper subgroup of s_3 is cyclic.

4) Let H be a subgroup of a group G then prove that

i) $xH=yH$ iff $x^{-1}y \in H$ ii) $Hx=Hy$ iff $xy^{-1} \in H$

5) Let H be a subgroup of group G & then prove that HUk is a subgroup of G iff either $H \subseteq K$ or $K \subseteq H$.

6) i) Let Hbe a subgroup of group G & $x,y \in G$ then prove that $xH=H$ iff $x \in H$.

ii) Let H,K be subgroup of group G then prove that $H \cap K$ is also a subgroup of G.