

- Instructions:** 1) All questions are compulsory.
2) Figures to right Indicate full marks.
3) Illustrations, in-depth answers and diagram will be appreciated.

Q.1 Attempt all (5marks each)

(15)

A) Multiple choice questions.

i) Which of the following commands will create a list?

- a) list1=list() b) list1=[] c) list1=([1,2,3]) d) All of these.

ii) The dot product of (1,3,1) & (0,1,-1) is _____

- a) 2 b) -2 c) 1 d) None of these.

iii) The dot product of (0,1,4) & (1,2,3) is _____

- a) -10 b) 10 c) 12 d) None of these.

iv) A vector whose norm is called _____

- a) Null b) basis c) Unit d) None of these.

v) For any homogenous system _____ is a trivial solution

- a) zero b) nonzero c) one d) None of these.

B) Fill in the blanks

i) The output when we execute list ("Hello") is _____

ii) If diagonal entry of square matrix is one & non-diagonal entry is zero then matrix is called. _____

iii) To add a new element to a list we use _____ command.

iv) The absolute value of Hui is _____

v) Inverse of a matrix is _____

C) Answer the following question.

- 1) Define dot product
- 2) Define dimension
- 3) Find dot product of (1,5), (4,-2)
- 4) Solve (1.1) + (0.1) + (0.1)
- 5) Determine the term characteristic equation.

Q.2 Attempt the following (Any Three)

(15)

- a) Find the square root of complex number $7-4i$
- b) Determine whether $v_1=(2,1,2)$, $v_2=(0,1,3)$, $v_3=(1,1,1)$ span vector space \mathbb{R}^3
- c) Write a python program to find conjugate of complex number.
- d) Are the following vectors linearly dependent?
 $v_1=(3,1,3)$, $v_2=(2,4,6)$, $v_3=(1,-1,6)$
- e) Express in polar and exponential form $1-i\sqrt{3}$
- f) Check whether the set of all pairs of real numbers of the form $(1, x)$ with operation $(1, y) + (1, y') = (1, y + y')$ and $k(1, y) = (1, ky)$ is a vector space.

Q.3 Attempt the following (Any Three)

(15)

- a) Find the angle between the two vectors $a = (2,3,4)$, $b = (1, -4,3)$ in \mathbb{R}^3
- b) Find null space in matrix $\begin{bmatrix} 1 & 3 & 1 \\ 2 & 4 & 3 \\ 1 & 1 & 2 \end{bmatrix}$
- c) Let $f: U \rightarrow V$ be linear transformation then show that $\ker f = \{0\}$ iff f is injective.
- d) Consider subspace $U_1 = \{(x, y, w, z) : x - y = 0\}$ and $U_2 = \{(x, y, w, z) : x = w, y = z\}$
find basis & determination of i) U_1 ii) U_2 iii) $U_1 \cap U_2$
- e) Check whether the set of functions are linearly independent $2 - x + 4x^2, 3 + 6x + 2x^2, 2\log x - 4x^2$
- f) If V, W are two subsets of a vector space V such that U is a subset of W then show that W^0 is a subset of U^0 where U^0, W^0 are annihilator of U, W respectively.

Q.4 Attempt the following (Any Three)

(15)

- a) Find orthonormal basis for subspace \mathbb{R}^4 whose generators are $v_1=(1,1,1,1)$,
 $v_2=(1,2,4,5)$, $v_3=(4,3,-4,-2)$
- b) Let $a=(3,0)$, $b=(2,1)$ find vector in span $\{a\}$ that is closest to b is b''^a and distance $\|b - a\|$
- c) Find inner product, angle, orthogonality for $p = -5 + 2x - x^2$, $q = 2 + 3x^2$
- d) Write program in python to find g.c.d(240,36)
- e) Let u, v be orthogonal vectors then prove that scalar a, b $\|au + bv\|^2 = a^2\|u\|^2 + b^2\|v\|^2$
- f) Explain internet worm.

(15)

Q.5 Attempt any three of following.

a) Let $T: \mathbb{R}^3 \rightarrow \mathbb{R}^2$ be linear map be defined by $f(x, y, z) = (x + 2y - z, x + y - 2z)$ verify (Rank T + Nulity $T=3$).

b) Find eigen values & eigen vectors of $\begin{bmatrix} 3 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & -1 & 3 \end{bmatrix}$

c) Let S be a subset of vector space V . prove that S^\perp is a subspace of V .

d) Express the following as linear combination of $V_1=(-2,1,3)$ $V_2=(3,1,-1)$, $V_3=(-1,-2,1)$ with $w=(6,-1,5)$.

e) Fill the table vector

space	basic	dimension
$\{0\}$	_____	_____
\mathbb{R}^2	$\{(1,0),(0,1)\}$	_____
$P_2(x)$	_____	3
$M_2(\mathbb{R})$	_____	4
\mathbb{R}	$\{1\}$	_____