4/10/2017

VCD- S.Y.B.Sc MATHS-I IV-SEMESTER EXAM.- 2016-17 75-MARKS 2½-HRS. PG-2

Instructions:- 1) All questions are compulsory.

2) Figures to the right indicate the marks.

Q. 1 A) Attempt any one 8

1) Define Iterated limits of a function.

Let f : $IR^2 \to IR$ given by $f(x, y) = \frac{x^2}{x^2 + y^2 - x}$, where $x^2 + y^2 - x \neq 0$.

show that the iterated limits of f are exists and equal, But $\lim_{(x, y) \to (0, 0)} f(x, y)$ does not exist.

2) Let $\phi \neq E \subseteq IR^n$ (n>1) and $P \in IR^n$ be a limit point of E. Let F : $E \to IR^m$ and g : $E \to IR^m$ (m > 1) be a functions such that $\lim_{x \to P} f(x) = l$ and $\lim_{x \to P} g(x) = m$ then show that $\lim_{x \to P} \langle f(x), g(x) \rangle = \langle l, m \rangle$

B) Attempt any three 12

1) Prove that $S = \{(x, y) \in IR^2 / x > 0$ is an open set, using definition of an open set.

2) Using definition of limits, Prove that

$$\lim_{(x, y) \to (2, 1)} x + 2y = 4$$

3) Define convergence of a sequence in IR? Using it prove that

$$\lim_{n \to \infty} \left(\frac{1}{n}, 1 + \frac{1}{n}\right) = (0, 1)$$

4) Define directional derivative of a function find directional derivative of $f(x, y) = 3x + 4y^2$ at (3, 1) in a direction of u = (–2, –2)

Q. 2 A) Attempt any one 8

1) Show that, partial derivative function

$$f(x, y) = \frac{xy}{x^2 + y^2}, \quad (x, y) \neq (0, 0)$$

= 0 , otherwise , at (0, 0) exist but f is not differentiable at (0, 0)

2) State and prove Mean Value theorem for function of n variable.

B) Attempt any three 12

1) Find equation of tangent plane and normal line of the surface $yz = \log(x + z)$ at (0, 0, 1)

2) Let E be an open subset of IR^n and $f: E \to IR$ be a differentiable at $P \in E$, then for any unit vector $U \in IR^n$, Duf(P) exist and Duf(P) < f(P), = < $\nabla F(P)$, U>

3) If $f: IR^n \to IR$ and g : $IR^n \to IR$ are differentiable at $P \in IR^n$, then f + g is also differentiable at P.

4) Let E be an open subset of IR^n. Let $f: E \to IR$ be a differentiable at $P \in E$, then f is continous at P.

Q. 3 A) Attempt any one

1) State and prove Mean Value inequality for vector field.
2) Find the point on ellipse $x^2 + 2y^2 = 1$ where $f(x, y) = xy$ has it's extreme value.

B) Attempt any three

1) Define Jacobian matrix of vector valued function. Find Jacobian matrix of $f(x, y) = (x \cos y, y \sin x)$ at $\left(\pi/4, \pi/4\right)$
2) Prove that derivative of vector valued function is unique, if exist.
3) Using Taylor's theorem, Expand the function $f(x, y) = \sin x \sin y$ at $(0, 0)$, upto 2^{nd} degree term.
4) Define Linear approximation. Find linear approximation of $f(x, y) = ln\,(x - 3y)$ at $(7, 2)$ Use it to approximate $f(6.9\ , 2.06)$

Q. 4 Attempt any three

1) Using 2^{nd} derivative test Examine the function $f(x, y) = x^2 - 4xy + y^2 + 6y + 2$ for extreme values.
2) Define Hessian matrix. Find Hessian matrix of the function $F(x, y, z) = x^3 + 2xyz + y^2z$ at $(1, 1, 1)$
3) Let $f : IR^2 \to IR^2$ be a function given by $f(x, y) = (|x| + |y|, xy)$. Using $\epsilon - \delta$ definition show that if f is continuous at $(0, 0)$ by proving that component functions are continuous at $(0, 0)$
4) Define Partial derivative of function. Find partial derivative of $f(x, y) = 4x^2 + 3xy + y^2 + 8x + y$ at $(0, 0)$
5) State and prove chain rule for derivative of scalar field
6) If $z = \dfrac{xy}{x^2 + y^2}$ where $(x, y) \neq (0, 0)$ and $x = r\cos\theta$ and $y = r\sin\theta$. then show that $\dfrac{\partial z}{\partial r} = 0$ and $\dfrac{\partial z}{\partial \theta} = \cos 2\theta$

—— The End ——