VCD/ 7|10|22 SYBSC- SEM III MATHEMATICS- I 75MARKS 2½HRS

NOTE :

1. **All questions are compulsory.**
2. **For Q.1, Q.2 and Q. 3 attempt any one sub question (each 8 marks) from part (a), and any two sub questions (each 6marks) from part (b).**
3. **For Q.4 , attempt any three. (each 5 marks)**

Q.1. (a) Attempt any one. [each 8Mks]

1) Define convergent and divergent series and Prove that Σar^{n-1} for n=1 to ∞ the geometric series converges iff $|r| < 1$
2) State and prove Leibniz test for an alternating series.

(b) Attempt any two. [each 6Mks]

1) Investigate the convergence of

$$\sum_{n=1}^{\infty} \frac{\cos n\pi}{n\sqrt{2}}$$

2)Prove that if Σa_n is a convergent series of R then lim a_n as n tends to ∞ is equal to 0

3)Prove that if Σa_n ,Σb_n are two convergent series then prove that

$\Sigma(a_n+b_n)$, $\Sigma(a_n-b_n)$is convergent series.

Q.2. (a) Attempt any one. [each 8Mks]

1) Let f:[a, b]→R be bounded function and P be partition of [a, b] then prove that

i) $m(b-a) \leq L(P, f) \leq M(b-a)$

ii) $m(b-a) \leq U(P, f) \leq M(b-a)$where m, M are Infimum and supreme of f(x) over [a, b] respectively

2)Let f:[a, b]→R be bounded function then prove that f is integrable iff for every $\epsilon>0$ there exist a partition P of [a, b] such that $U(P, f) - L(P, f) < \epsilon$.

(b) Attempt any two. [each 6Mks]

1) Prove that If f is integrable on [a, b] then f^2 is also integrable on [a, b]
2) Define L(P, f) , U(P, f) where P is partition of [a, b] for bounded function f:[a, b]→R and prove that for bounded function f:[a, b]→R , Partition P of [a, b] , $L(P, f) \leq U(P, f)$

3) Prove that a constant function is Riemann integrable.

Q.3. (a) Attempt any one. [each 8Mks]

1) Let a>0, then prove that $_a\int^{\infty} dt/t^P$ converges iff P>1.

2) Define Beta function β(m, n) and prove that β(m, n)=β(n, m) .Define Gamma function and prove that Γ(n+1)= n Γ(n)for n>0

(b) Attempt any two. [each 6 Mks]

1) i) Define Indefinite integral of f on [a, b] Find indefinite integral of f on [0,1] for f:[0,1]→R as $f(x) = 2x^3+3$.

ii) Define Primitive if function f. Give an example to show that primitive of f if exist,need not be unique.

2)Prove that Let f:[a, b]→R be a continuous function then there exist c∈[a, b] such that $_a\int^b f(x)\, dx = f(c)\,(b-a)$

3)State Leibnitz rule and Use it to find the derivative of the following $f(x) = {}_{\cos x}\int^{\sin x} \frac{1}{1-t^2}\, dt$

Q.4.Attempt any three. [each 5 Mks]

1) Discuss the convergence of following series

$$\sum_{n=1}^{\infty} \frac{(-1)^n - 6^n}{15^{n+2}}$$

2) Find the sum of the series

$$\sum_{n=1}^{\infty} \frac{20}{(5n-4)(5n+1)}$$

3) Let f:[0,1]→R as f(x) =3x Prove that f is integrable and find $_0\int^1 3x\, dx$.
4) Find the norm of following partition of intervals I =[-2,5], P={-2,-1.5,-1.25,0,2.6,,2.9,4.3,5}.
5) Find the value of c in [0,1] such that $f(c)=f_{avg}$ for $f(x)=5x^4$, x∈[0, 1]
6) Find the area between the curves $y=2x^2$ and y=x between x=0 and x=2.

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