VCD/ 101022 SYBSC- SEM III MATHEMATICS- II 75MARKS 2½HRS

NOTE : 1)All questions are compulsory.

2)For Q.1, Q.2 and Q. 3 attempt any one subquestion (each 8 marks) from part (a), and any two subquestions (each 6marks) from part (b).

3)For Q.4 , attempt any three. (each 5 marks)

Q.1. (a) Attempt any one. [each 8Mks]

1)Prove that row equivalence relation on the set of m×n matrices.

2)Let A, B be the matrices of order m×n then prove that A, B are row equivalent off there existsan invertible matrix P such that B=PA

(b) Attempt any two. [each 6Mks]

1) Check whether the following system of equation is consistent and if so, find the solution set. $3x+7y-5z=6$

$$x+4y+3z=7$$

$$4x+7y-z=20$$

2) Check whether the following matrices are elementary matrices $i)\begin{bmatrix}0 & 1 & 1\\0 & 1 & 0\\1 & 0 & 0\end{bmatrix} ii)\begin{bmatrix}1 & 2 & 3\\0 & 1 & 0\\ & 1 & \end{bmatrix} iii)\begin{bmatrix}1 & 1\\-2 & 1\end{bmatrix}$

3) Express the following matrices and their inverses as a product of elementary matrices. $\begin{bmatrix}9 & 11\\6 & 2\end{bmatrix}$

Q.2.Attempt any one. [each 8Mks]

1)Let V be a real vector space and W be a subspace of V then prove that

i) v+W=W iff v belongs to W ii) v+W=v'+W iff v-v' belongs to W

2)Define Linearly independent set and Prove that Subset of linearly independent set is linearly independent.

(b)Attempt any two. [each 6Mks]

1) Check whether the given vector is belong to L(S) for v=(1, 3,2), S={(2,1,-1),(3,2,0),(4,0,3)} in R^3

2) Prove that the set of all points on the plane ax+by+cz=0 passing through origin form a subspace in R^3 with respect to usual addition and scalar multiplication.

3) Let (V, +,.) Be a vector space and W_1, W_2 be two subspace of V then prove that $W_1 \cap W_2$ is also a subspace of V.

Q.3. (a) Attempt any one. [each 8Mks]

1) Let A be A n×n real matrix.If $\det(A) \neq 0$ then A is invertible and $A^{-1} = (1/\det(A)) \times (\mathrm{Adj}(A))$

2) Prove that $\det(V_n) = \det(V_n^T) = \prod_{1 \leq i < j \leq n} (x_i - x_j)$

(b) Attempt any two. [each 6 Mks]

1) Prove that determinant of upper or lower triangular matrix is product of diagonal entries.

2) Find the adjoint of the following matrix $\begin{bmatrix} -9 & 1 & 6 \\ 7 & 5 & 4 \\ -6 & 1 & 3 \end{bmatrix}$

3) Find row rank, column rank and rank of the following matrix using row echelon form also find basis of row space and column space. $\begin{bmatrix} 0 & 2 & 6 & 5 & 9 \\ 1 & 2 & 3 & 6 & 2 \\ 2 & 4 & 0 & 1 & 0 \end{bmatrix}$

Q.4. Attempt any three. [each 5 Mks]

1) Show that the following system of equations have infinitely many solutions applying Gauss Elimination method $2x-y=10, 10x-5y=50$

2) Express the following matrix in row echelon form in two different way. $\begin{bmatrix} 4 & 1 \\ 6 & 3 \end{bmatrix}$

3) Prove that $(\mathbb{R}^2,+,.)$ Is a vector space with respect to addition and scalar multiplication in $\mathbb{R}^2$ for + and. As follows $(a, b) + (c, d) = (a+b, c+d)$ and $\alpha(a, b) = (\alpha a, \alpha b)$ for $\alpha \in \mathbb{R}$ and $(a, b), (c, d) \in \mathbb{R}^2$

4) Find the basis and dimension of $W = \{(x_1, x_2, x_3) \;/\; x_1 - 2x_2 + 3x_3 = 0\}$ of $\mathbb{R}^3$

5) Find the determinant of following matrix. $\begin{bmatrix} -9 & 3 & 3 & 3 \\ 3 & -9 & 3 & 3 \\ 3 & 3 & -9 & 3 \\ 3 & 3 & -9 & 3 \end{bmatrix}$

6) Find the nulity of following coefficient matrix for given system of equation.

$x_1+4x_2+x_3=2, \quad x_1+2x_2-x_3=0, \quad x_1+6x_2=3$

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