VCD/ 121022 SYBSC- SEM III MATHEMATICS - III 75 MARKS $2\frac{1}{2}$ HRS

Note: (i) All questions are compulsory.
(ii) Figures to the right indicate marks.

1. Attempt any **ONE** question from the following : **(08)**

a) i. The substitution $x = e^z$ reduces the Cauchy equation $x^3\frac{d^3y}{dx^3} + px^2\frac{d^2y}{dx^2} + qx\frac{dy}{dx} + ry = 0$ into the differential equation $\frac{d^3y}{dx^3} + (p-3)\frac{d^2y}{dz^2} - (p-q-2)\frac{dy}{dz} + ry = 0$ and $y = x^m$ is a solution to the given Cauchy equation if and only if m is the root of the equation $m^3 + (p-3)m^2 - (p-q-2)m + r = 0$.

ii. Let $y_1(x)$ *and* $y_2(x)$ be two solutions to the differential equation to $y'' + P(x)y' + Q(x)y = 0$ Where $P(x), Q(x)$ are continuous on an interval (a, b) then y_1 *and* y_2 are linearly dependent on (a. b) $W(y_1, y_2)x = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = y_1y_2' - y_2y_1' = 0 \;\forall\; x \in (a, b)$

b) Attempt any **TWO** questions from the following: **(12)**

i. Solve the differential equation: $((2x-1)^3D^3 + (2x-1)D - 2)y = 0$

ii. Find the particular solution to the differential equation $(D^3 + 2D^2)y = 0$; When $x = 0,\ y = -3,\ y' = 0,\ y'' = 12$

iii. Verify $y_1 = e^{2x}$ and $y_2 = e^x$ are solution to differential equation $y'' - 3y' + 2y = 0$. Also verify $y = C_1y_1 + C_2y_2$ is the solution of any two real number $C1, C2$. Further find $C1, C2$ using initial condition $y(0) = 1, y'(0) = -1$.

2. Attempt any **ONE** question from the following : **(08)**

a) i. Define Wronskian. If $x = x_1(t), y = y_1(t)$ *and* $x = x_2(t), y = y_2(t)$ are two linearly independent solutions to the homogeneous linear system $\frac{dx}{dt} = a_1(t)x + b_1(t)y, \frac{dy}{dt} = a_2(t)x + b_2(t)y$ then in [a, b] the general solution to above system is $x = c_1x_1(t) + c_2x_2(t)$, $y = c_1y_1(t) + c_2y_2(t)$ where c_1, c_2 are arbitrary constants.

ii. Obtain the general solution of a Homogeneous Linear system (of two equations) with constant coefficients, when its Auxiliary equation has two real and equal roots.

b) Attempt any **TWO** question from the following: **(12)**

i. Using method of variation of parameter find a particular solution $\frac{dx}{dt} = -2x + y, \frac{dy}{dt} = -3x + 2y + 2sint$ and hence find the general solution.

ii. Find the general solution to the linear system $\frac{dx}{dt} = 4x - 3y, \frac{dy}{dt} = 8x - 6y$

iii. Show that the solutions $x = e^{4t}, y = e^{4t}$ *and* $x = e^{-2t}, y = -e^{-2t}$ are linearly independent of the system of equation $\frac{dx}{dt} = x + 3y, \frac{dy}{dt} = 3x + y$

3. Attempt any **ONE** question from the following : **(08)**

a) i. State the procedure of Picard's method and hence using it find 4th approximation polynomial $y_4(x)$ of the differential equation $y' = 2x + y - 2$ *with* $y(0) = 2$

ii. Solve the system of ordinary differential equations $\frac{dx}{dt} = x + y - t, \frac{dy}{dt} = 2x - y + 3t$ $with\ x = 1, y = 1 when\ t = 0, taking\ \Delta t = h = 0.1$

b) Attempt any **TWO** question from the following: **(12)**

a) i. Using Euler's Modified method solve $y' = 4x + 9y$, $y(1) = 0.5, h = 0.25$ correct upto 4 decimal places for finding y(1.25)

ii. Using Taylor's method find the polynomial of degree 4 that satisfies the differential equation $y' = 2y^2 - x^2, y(1) = 1.5$ and using this find the approximation value of $y(1.1)$

iii. Use Runge-Kutta method of 4th order to estimate y(1.25) when $y' = 2x^3y - 3, y(1) = 1.3\ and\ h = 0.25$

4. Attempt any **THREE** question from the following : **(15)**

a) Solve the following differential equation $y'' + 9y = 12sin2x$

b) Find the general solution to the following differential equation $x^2y'' + 3xy' + 10y = 0$

c) Reduce the differential equation $4y'' - 7y' + 6y = 0$

d) Define a system of homogeneous linear differential equations of order1. State the condition for two solutions (x_1, y_1) and (x_2, y_2) to be linearly independent. Also write the general solution.

e) Estimate y(0.25) using Runge-Kutta method of 2nd order with $w_1 = \frac{1}{3}, w_2 = 2/3$ for the differential equation $y' = \frac{x-y}{2}, y(0) = 1\ and\ h = 0.25$

f) Using Euler's method solve $y' = 5x^2 + 2xy$, $y(1) = 0.25, h = 0.15$ for finding y(1.6).

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