19/10/19

VCD SYPM Sem-III Subject- Discrete Mathematics Paper-III Exam 2019

(3 Hours)

[Total Mark: 100]

Note: (i) All questions are compulsory.

(ii) Figures to the right indicate marks for respective parts.

1. Choose correct alternative in each of the following : (20)

i) The number of elements in A_6 is

a) 6
b) 720
c) 360
d) 2^6

ii) The recurrence system with the initial condition $a_0 = 0$ and recurrence relation $a_n = a_{n-1} + 2n - 1$ is linear of

a) Degree 2 and non homogeneous
b) Degree 1 and non homogeneous
c) Degree 1 and homogeneous
d) None of these

iii) For the sequence $a_n = 6(1/3)^n$, then a_4 is

a) $2/25$
b) $2/27$
c) $2/19$
d) $2/13$

iv) Let S(n, k) denote stirling number of second kind on n-set into k disjoint non empty ordered subset then S(0, 0) is

a) 1
b) 0
c) n
d) None of these

v) The number of functions from a set with m elements to one with n elements are

a) m^n
b) n^m
c) $m \times n$
d) None of these

vi) The Cartesian product of two countable sets A and B is

a) Countable
b) Uncountable

c) Finite

d) None of these

vii) A student can choose a computer project from one of three lists. The three lists contain 23, 15 and 19 possible projects respectively. How many possible projects are there to choose from

a) 38

b) 57

c) 34

d) 42

viii) $\varphi(13)$ is

a) 13

b) 12

c) 14

d) None of these

ix) How many solutions are there to the equation $x_1 + x_2 + x_3 + x_4 = 17$ have, where x_1, x_2, x_3 and x_4 are non negative integers ?

a) C(20,3)

b) C(17,3)

c) C(21,3)

d) None of these

x) At a party there are n men and n women. In how many ways can the n women choose male partners for the dance?

a) D_n

b) $n!$

c) $(n-2)!$

d) None of these

2. Attempt any **ONE** question from the following : (08)

a) i. Show that if the characteristic equation $x^2 - r_1 x - r_2 = 0$ of the recurrence relation $a_n = r_1 a_{n-1} + r_2 a_{n-2}$ has a single root then the explicit formula is $a_n = us^n + vns^n$.

ii. Prove that product of two disjoint cycles is commutative.

b) Attempt any **TWO** question from the following : (12)

i. Define an even permutation. Express $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 8 & 2 & 6 & 3 & 7 & 4 & 5 & 1 \end{pmatrix} \in S_8$ as a product of disjoint cycles. Determine whether σ is odd or even.

ii. Define signature of a permutation. If σ is any permutation in S_n then show that the sign of σ is ± 1.

iii. Solve the recurrence relation $a_n = 5a_{n-1} - 6a_{n-2};\ a_0 = 1, a_1 = 0$

iv. A bank pays 8% interest each year on money in the savings account. Find the recurrence relation for the amounts a person would have after n years if it follows the investment strategy of
(a) investing 1000 and leaving it in the bank for n years.
(b) investing 1000 at the end of each years.

3. Attempt any **ONE** question from the following : (08)

a) i. Show that the interval $[0,1]$ is uncountable.

ii. Define Stirling number $S(n,k)$ of second kind. Prove that
$S(n, n-2) = \binom{n}{3} + 3\binom{n}{4} = \frac{1}{4}(3n-5)\binom{n}{3}$

b) Attempt any **TWO** question from the following : (12)

i. Let n and k be positive integers. Show that the number of surjective functions from an n- set to a k-set is equal to $k!\,S(n,k)$ where $S(n,k)$ *is* Stirling number of second kind.

ii. State Pigeonhole principle. There are 60 rooms and 1000 students of particular class in a college. Show that at least one class has at least 17 students.

iii. State Addition and Multiplication principle. How many different four letters initials can people have? Also find how many of them have no repetition in their initials?

iv. How many different 4- letter radio station call letters (upper case) can be made
a) if the first letter must be a K or W and no letter may be repeated.
b) if repeats are allowed (but the first letter is a K or W).
c) How many of the 4- letter call letters (starting with K or W) with no repeats end in R?

4. Attempt any **ONE** question from the following : (08)

a) i. Prove by giving combinatorial argument: $\sum_{k=0}^{r}\binom{m}{k}\binom{n}{r-k} = \binom{m+n}{r}$

ii. Define Euler$\emptyset$ function. Let $n \geq 2$ be an integer whose prime factorization is

$n = {p_1}^{e_1} {p_2}^{e_2} \dots . {p_r}^{e_r}$ where $e_i \geq 1, \forall i, 1 \leq i \leq r$, prove that

$\emptyset(n) = n\left(1 - \frac{1}{p_1}\right)\left(1 - \frac{1}{p_2}\right) \dots \dots \left(1 - \frac{1}{p_r}\right)$

b) Attempt any **TWO** question from the following : (12)

i. State and Prove Binomial Theorem.

ii. Let S be a multiset with objects of k different types each with an infinite repetition number (multiplicity). Show that the number of r- combinations of S equals

$$\binom{r+k-1}{r} = \binom{r+k-1}{k-1}$$

iii. Define Derangement D_n. Show that $D_n = nD_{n-1} + (-1)^n$, n≥ 2 with $D_1 = 0$ and $D_2 = 1$.

iv. Find the coefficient of x^3yz^2 in the expansion of $(2x - 3y + 5z)^6$. Also find the numbers of terms and sum of all the coefficients in the expansion.

5. Attempt any **FOUR** question from the following : (20)

a) Define Even permutation. Prove that product of two even permutation is even.

b) Solve the linear non homogeneous recurrence relation $a_n = 3a_{n-1} + 2n;\ a_0 = 3$

c) Prove by Mathematical induction $S(n, n-1) = \binom{n}{2}, n \geq 2$

d) Prove that if seven distinct numbers are selected from $\{1,2, \dots, 11\}$, then two of these numbers sum to 12.

e) How many 11 letter words can be made from the letters of the word MISSISSIPPI?

f) How many solutions does the equation $x_1 + x_2 + x_3 + x_4 = 20$ have, in which $x_1 \geq 3, x_2 \geq 1, x_3 \geq 0, x_4 \geq 5$?

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