VCD - 10/10/17 - MATHS III - SYBSC - SEM III EXAM - 75 MARKS - $2\frac{1}{2}$HRS -

Note: : 1) All questions are compulsory.

2)For Q.1, Q.2, Q.3, attempt any one subquestion (each 8 mks)from part (a), and any three subquestions (each 4 mks) from part(b)

3) For Q.4 Attempt any three.(each 5 mks)

Q.1 (a) Attempt any one [Each 8]

1) Prove that N x N is Countable.

2) Prove that [0,1] ~ [0,1].

Q.1 (b) Attempt any three. [Each 4]

1) How many different subsets of N_{10} having exactly 5 numbers and each number from N_{10} belongs to exactly 4subsets. Write all these subsets.

2) Verify Explicit formula for n = 10, k = 4 where S (10,4) = 34105.

$$S(n,k) = \frac{1}{k!}\sum_{j=0}^{k}(-1)^j \, {}^{k}C_{j} \, (k-j)^n$$

3) Prove that,S(n,2) =$2^{n-1} - 1$; $n \geq 2$

4) A train required 15 hours to complete the journey of 972 kms. From Pune to Indore. It is known that the speed of the train was 50kms/hr in first 3 hours and 40kms/hr in last 3 hours. Show that the train must have travelled at least 234 kms . Within a certain period of three consecutive hours.

Q.2 (a) Attempt any one [Each 8]

1) How many solution does x + y + z =11 where x, y and z are non-negative integers with $x \leq 3$, $y \leq 4$, $z \leq 6$.

2) Let n be a positive integer such that

$$n = p_1^{e1} \, p_2^{e2} \, p_3^{e3} \cdots\cdots p_r^{er}$$ where $p_1 p_2 p_3 \ldots\ldots\ldots\ldots p_r$ are distinct

primes and $e_1 e_2 e_3 \ldots\ldots\ldots\ldots e_r \geq 1$ Then the Euler Phi function ø (n) is given by

$$ø(n) = n \prod_{i=1}^{r}\left(1 - \frac{1}{p_i}\right)$$

Q.2 (b) Attempt any three. [Each 4]

1) find the number of positive integers from 1 to 250 which are not divisible by 2 or 3 or 5. Also find how many integers are divisible by 2 not by 3 or 5

2) Find ø (60) using Inclusion-Exclusion Principle.

3) Show that if n is a positive integer then prove that

$$\binom{2n}{2} = 2\binom{n}{2} + n^2$$

4) Let n and k be positive integers with $k \leq n$ then,

$$\overset{n+1}{\underset{k}{C}} = \overset{n}{\underset{k-1}{C}} + \overset{n}{\underset{k}{C}}$$

Q.3 (a) Attempt any one [Each 8]

1) For ά, ß Ɛ S_n. Prove that Sign(ά ο ß) = Sign(ά).Sign(ß).

If $ά = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 5 & 2 & 6 & 7 & 3 & 1 & 4 \end{pmatrix}$, $ß = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 5 & 6 & 3 & 7 & 1 & 4 & 2 \end{pmatrix}$,

Find Sign of ά, ß and (ά ο ß).

2) i) If the characteristics equation $x^2 - r_1x - r_2 = 0$ of the recurrence relation $a_n = r_1 a_{n-1} + r_2 a_{n-2}$ has two distinct roots s_1 and s_2 then prove that $a_n = us_1^n + vs_2^n$ where value of u and v depends on initial conditions, is the explicit formula for the sequence.

ii)Give first five terms of given recurrence relation. $b_n = -3b_{n-1} - 2\, b_n - 2$, $b_1 = -2$, $b2 = 4$

Q.3 (b) Attempt any three. [Each 4]

1) Solve the following recurrence relation

i) $a_n = 3a_{n-1}, n \geq 2, a_1 = 3$

2) Define the signature of a permutation and find the signature of $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 1 & 3 \end{pmatrix}$

3) i) Write down $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 5 & 1 & 2 & 4 \end{pmatrix}$ in cyclic form also as a product of Transposition.

ii) Write down (1 2)(3 5) cycle of S_5 in the standard form

iii) Find the product $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 5 & 1 & 4 \end{pmatrix}\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 5 & 2 & 3 & 4 \end{pmatrix}$

iv) Find invers of $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 5 & 3 & 1 & 4 \end{pmatrix}$

4) Prove that For any integer $n \geq 2$ exactly half of permutations in S_n are odd and half are even.

Q.4 Attempt any three [Each 5]

1) Prove that Set of Positive Rational Numbers Q^+ is Countable.

2) Find the Coefficient of x^5 in expansion of $(1 + 3x + 2x^2)^4$.

3) Use Pascal's Identity to Prove that $\sum_{k=0}^{r} \binom{n+k}{k} = \binom{n+k+1}{r}$

Whenever n and r are positive integers.

4) If n and r are integers with $0 \leq r \leq n$

Then $^{n}P_{r} = \frac{n!}{(n-r)!}$

5) Solve the recurrence relation

$$d_n = -3d_{n-1} - 3d_{n-2} - d_{n-3}, d_1 = d_2 = 3, \quad d_3 = 7.$$

6) Define the terms

1. Recurrence relation 2. Linear Recurrence relation
3. Linear homogeneous Recurrence relation
4. Linear Non-Homogeneous Recurrence relation