VCD -7|10|17- MATHS I - SYBSC - SEM III EXAM - 75 MARKS - 2.5HRS

Note: : 1) All questions are compulsory.

2)For Q.1, Q.2, Q.3, attempt any one subquestion (each 8 mks)from part (a), and any three subquestions (each 4 mks) from part(b)

3) For Q.4 Attempt any three.(each 5 mks)

Q.1 (a) Attempt any one [Each 8]

1) If f and g are Riemann integrable on I then f+g is also Riemann integrable on I. and $\int_a^b f + g = \int_a^b f + \int_a^b g$.

2) State and prove Riemann criterian for integrability of function f on [a,b].

Q.1 (b) Attempt any three. [Each 4]

1) Define upper integral and lower integral.
If f: I → IR is bounded and P is any partition of I,then m(b-a) ≤ U(f)≤ M(b-a).

2) If f is Riemann integrable on I then λf is also Riemann integrable on I and $\int_a^b \lambda f = \lambda \int_a^b f$.

3) Define upper sum and Lower sum.
If f: I → IR is bounded and P is any partition of I,then L (P,f)≤ U(P,f).

4) If f is integrable on [a,b] then IfI is also integrable on [a,b] and $|\int_a^b f| \leq \int_a^b |f|$

Q.2 (a) Attempt any one [Each 8]

1) State and prove Leibnitz rule. Using it evaluate $\frac{d}{dx}\int_1^{x^3} \frac{1}{t^4+4}$dt.

2) Define Beta and Gamma function. State and prove Beta-Gamma relation.

Q.2 (b) Attempt any three. [Each 4]

1) Define Improper integral of type 2. Evaluate $\int_0^1 \frac{1}{x^{\frac{1}{2}}(1+x^2)}$ dx.

2) Prove that $\int_a^\infty \frac{1}{x^p} dx$,(a>0) converges if and only if p ≥ 1.

3) State the comparision test for type 1 improper integral. Examine the convergence of $\int_1^\infty \frac{\sin^2 x}{x^2}$ dx using it.

4) prove that $\lceil n = (n-1)\lceil\overline{n-1}$ =(n+1)!.

Q.3 (a) Attempt any one [Each 8]

1) Find mass and Centre of Mass of the region bounded bythe lines y=x , y= -x, and y=1, ifdensity function $\delta(x,y)=3x^2+1$.

2)Derive the formula for double integration over the rectangular region.

Evaluate $\int_0^1 \int_1^2 \frac{xe^x}{y}\,dy\,dx$.

Q.3 (b) Attempt any three. [Each 4]

1)Evaluate the double integration $\int_0^2 \int_0^{\sqrt{(4-y^2}} (x^2+y^2)\,dy\,dx$ using polar coordinate system.

2)Express r = 1+cosθ into Cartesian coordinate system.

3) Find the area of the region bounded by $y = x^2$ and $y = x^3$.

4) Find the Volume of the Solid that lies under the hyperbolic praboloid $z=4+x^2-y^2$ and above the square R=[-1,1]x[0,2].

Q.4 Solve any three [each 5]

1) Find the Average height of the surface $z= 16-x^2-y^2$ over the square $0\leq x \leq 1$, $0 \leq y \leq 4$.

2) State the Fubini's theorem and Evaluate the double integration $\int_0^\pi \int_x^\pi \frac{\sin y}{y}\,dy\,dx$ by reversing the order of integration.

3) Prove that constant function is always Riemann integrable.

4) If a <c <b and f is Riemann integrable on [a,c] and on [b,c],then f is integrable on [a,b] and $\int_a^b f = \int_a^c f + \int_c^b f$.

5) Show that $\int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}}\,dx = \beta(m,n)$.

6) Evaluate $\int_{-1}^2 xe^{6x}\,dx$ using integration by parts.