FYBSC- SEM II -MATHS I - $2\frac{1}{2}$HRS - 75 MARKS - D: 23/9/2016

NOTE : 1) All questions are compulsory.

2) For Q.1, Q.2 and Q. 3 attempt any one subquestion (each 8 marks) from part (a), and any three subquestions (each 4 marks) from part (b).

3) For Q.4 , attempt any three.(each 5 marks)

Q.1. (a) Attempt any one. [each 8]

1) Let Σa_n, Σb_n be convergent series converging to a & b respectively then prove following

i) If $c_n = a_n - b_n \quad \forall\, n \epsilon \mathbb{N}$ then Σc_n is convergent, converging to $a - b$.

ii) If $\alpha \in \mathbb{R}$ is fixed and $c_n = \alpha\, a_n$ then Σc_n is convergent, converging to αa.

2) Examine convergence of following series.

i) $\Sigma \frac{n}{n^2 - cos^2(n)}$ ii) $\Sigma \frac{n^2+2}{n^4+5}$

(b) Attempt any three. [each 4]

1) Find n^{th} partial sum and determine if the series converge or diverge.

$$1 - 3 + 9 - 27 + \cdots + (-1)^{n-1}(-3)^{n-1}$$

2) State Leibnitz theorem and examine the convergence of $\sum_{n=2}^{\infty} \frac{\cos n\pi}{\sqrt{n}}$

3) Define an absolute convergence of series and prove that absolute convergent series are convergent.

4) State Modified root test and examine the convergence of $\sum_{n=1}^{\infty} (\frac{n+3}{n})^{n^2}$

Q.2. (a) Attempt any one. [each 8]

1) Define the derivative of a real valued function f at $x \in I$ an open interval subset of $\mathbb{R}$ and show that following functions f are differentiable on $\mathbb{R}$

i) $f: \mathbb{R} \to \mathbb{R}$ as $f(x) = c \quad c \epsilon \mathbb{R}$.

ii) $f: \mathbb{R} \to \mathbb{R}$ as $f(x) = a^x \quad a > 0$

2) State Leibnitz theorem and find n^{th} derivative of following

(b) Attempt any three. [each 4]

1) Find y_n, n^{th} order derivative of y if $y = \frac{1}{x^2-5x+6}$

2) If $xy - logy = 1$ then prove that $y^2 + (xy - 1)y_1 = 0$

3) Find equation of tangent and normal to the following

$x^2 + xy + 2y^2 = 28$ at (2,3)

4) If $-1 \leq x \leq 1$ then find derivative of $y = \sin^{-1} x$ $\quad -\frac{\pi}{2} < y < \frac{\pi}{2}$

Q.3. (a) Attempt any one. [each 8]

1) State Rolle's Theorem and give their geometrical interpretation.

Verify Rolle's Theorem for $\frac{x^2-4x}{x+2}$ in [0,4]

2) State Lagranges Mean Value Theorem and give their geometrical interpretation.

Verify Lagranges Mean Value Theorem for $f(x) = \sqrt{x^2 - 4}$ in [2,3]

(b) Attempt any three. [each 4]

1) Find the Taylor's Polynomials of degree n at $x = a$ for the mentioned values of a & n

$f(x) = \cos x \qquad a = \frac{\pi}{4}, n = 5$

2) State L-Hospital's Rule and evaluate following

$$\lim_{x \to 0} \frac{\log(1+x^3)}{\sin^3 x}$$

3) Find absolute maximum and minimum values of each function on the given interval. $f(x) = \frac{-1}{x^2}$ [0.5,2]

4) Divide the number 22 into two parts so that the sum of their square is minimum

Q.4. Attempt any three. [each 5]

1) Prove that the series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges if $p > 1$ and diverges if $p \leq 1$

2) Prove that the necessary and sufficient condition for a series Σa_n to be convergent is for any $\epsilon > 0$ $\exists n_0 \epsilon \mathbb{N}$ such that $\left|\sum_{p=n+1}^{m} a_p\right| < \epsilon \quad m > n \geq n_0$

3) If a function $f: I \to \mathbb{R}, I \subseteq \mathbb{R}$ open interval, is differentiable at $c \in I$ then prove that f is continuous at $c \in I$. What about converse?

4) Verify Cauchy Mean Value Theorem for $f(x) = \sqrt{x}, g(x) = \frac{1}{\sqrt{x}}$ in $[a, b]$

5) Sketch the graph of $y = x^3 - 3x^2 + 4$

6) Is $f(x) = |x|$ differentiable on $\mathbb{R}$?