

130522

VCD - \_\_\_\_\_ - FYBSC (PCM/PMS) SEM IIATKT-Algebra II-100MARKS - 3HRS

Q.1) Tick mark the correct option ( 2 mks each)

- 1) An elementary matrix of order  $n$  is matrix ....
  - A) obtained by applying only one row operation to identity matrix of order  $n$
  - B) obtained by applying only two row operation to identity matrix of order  $n$
  - C) obtained by applying only three row operation to identity matrix of order  $n$
  - D) obtained by applying only four row operation to identity matrix of order  $n$
- 2) A square matrix is invertible iff ....
  - A) it can be written as product of nonzero matrices
  - B) it can be written as product of nonzero and some zero matrices
  - C) it can be written as product of elementary matrices
  - D) it can be written as product of diagonal matrices
- 3) Row space of matrix  $[A]_{m \times n}$  is ....
  - A) subspace of  $R^n$  generated by row vectors of  $A$
  - B) subspace of  $R^n$  generated by column vectors of  $A$
  - C) subspace of  $R^m$  generated by row vectors of  $A$
  - D) subspace of  $R^m$  generated by column vectors of  $A$
- 4) The rank of null matrix is ....
  - A) 1 B) 0 C) 3 D) 2
- 5) State whether the following vectors are ...  $(1, 2, 3), (2, -2, 0)$ 
  - A) linearly independent B) linearly dependent
  - C) both A and B D) neither A nor B
- 6) A matrix unit is a matrix with dimensions .....
  - A) but with elements
  - B) but without actual elements
  - C) but all elements are zero
  - D) but all elements are one
- 7) \_\_\_\_\_ is invertible
  - A) No elementary matrix B) Every elementary matrix C) some of an elementary matrix D) any one of A, B
- 8) Let  $AX = B$  be a system of  $n$  linear equations in  $n$  unknowns. If  $|A| = 0$  then
  - A) system has unique solution
  - B) System has infinitely many solutions
  - C) System has no solutions
  - D) any one of B or C
- 9) In Gaussian Elimination method, to solve  $AX = D$ , the augmented matrix  $[A/D]$  is reduced to
  - A) identity matrix
  - B) null matrix
  - C) an upper triangular form

D) non singular matrix

10) Which of the following is false?

A) The homogeneous system of equations has a unique solution

B) The homogeneous system of equations has infinitely many solutions

C) The homogeneous system of equations has no solution

D) The homogeneous system of equations has zero solution

11) There are ----- types of elementary matrix

A) three types of an elementary matrix

B) An infinite types of an elementary matrix

C) only one type of an elementary matrix

D) as per order of the matrix

12) Let A be any nonsingular square matrix, B is a matrix of same order such that-----

A)  $AB=I$  B)  $BA=I$  C)  $AB=BA=I$  D)  $AB=BA=O$  where O is null matrix of same order

13) Matrix  $M_{m \times n}(R)$  is a vector space over \_\_\_\_

A) Z B) Q C) R D) N

14) Additive Identity in a vector space is \_\_\_\_

A) Not unique B) Unique C) 1 D) Either 1 or 0

15) Let V be a vector space and W be a non-empty subset of V. Then W is a subspace of V if and only if

A)  $a.x \in W$  B)  $a.x - b.y \in V$  C)  $a.x + b.y \in W$

D)  $a.x + b.y \in V$

16) Dimension of Vector space  $M_{2 \times 2}$  is,

A) 1 B) 2 C) 3 D) 4

17) Which of the following is not a subspace of  $R^3$ ?

A)  $S = \{(x, 0, 0) / x \in R\}$

B)  $W = \{(x, y, z) / z = x + y, x, y, z \in R\}$

C)  $P = \{(x, y, z) / x = y + z + 1\}$

D) Planes passing through origin in  $R^3$

18) Which of the following is a vector space?

A)  $S = \{(x, y, x + y) / x, y \in R\}$  in  $R^3$

B)  $S = \{(x, x^2) / x \in R\}$

C)  $S = \{(x, y, 2) / x, y \in R\}$  on  $R^3$

D)  $S = \{(a, b, c, a + 2) / a, b, c \in R\}$  in  $M_{2 \times 2}$

19) The \_\_\_\_ of two subspaces of a vector space V is always a subspace of V

A) Union B) Intersection C) Addition D) Product

20) Union of two subspaces of a vector space V is a subspace of V if and only if \_\_\_\_

A) They are disjoint B) One is a subset of other

C) Both are subsets of each other

D) Their intersection contains single element

21) \_\_\_\_ of finitely many subspaces of a vector space V is also a subspace of V.

A) Addition B) Union C) Intersection

D) Union as well as intersection



22) Which of the following set of matrices of real matrices of order 2 forms a subspace of  $M_2(\mathbb{R})$ ?

A) All skew symmetric matrices B) All invertible matrices C) All non-invertible matrices D) All matrices  $A$  such that  $A^2 = A$

23) Let  $V$  be a vector space, and let  $A$  &  $B$  be two subspaces of  $V$  such that either  $A \subseteq B$  or  $B \subseteq A$  then,

A)  $A \cup B$  is not a subspace of  $V$  B)  $A \cap B$  is not a subspace of  $V$  C)  $A \cup B$  is a subspace of  $V$  D)  $A \cup B$  may or may not be a subspace of  $V$

24) Inverse of a linear transformation....

A) need not be a linear transformation  
B) is again a linear transformation  
C) is not always linear transformation  
D) may not exist always

25) Find the linear transformation  $T$  from  $\mathbb{R}^2$  to  $\mathbb{R}^3$  such that  $T(1,0) = (4,3,-1)$  and  $T(0,1) = (-5,6,1)$  Find the linear transformation  $T$  from  $\mathbb{R}^2$  to  $\mathbb{R}^3$  such that  $T(1,0) = (4,3,-1)$  and  $T(0,1) = (-5,6,1)$

A)  $T(x, y) = (4x-5y, 3x+6y, -x+y)$   
B)  $T(x, y) = (4x-y, 3x+y, -x+y)$   
C)  $T(x, y) = (x-5y, x+6y, -x+y)$   
D)  $T(x, y) = (4x-5y, 3x+6y, -x-y)$

**Q.2) Attempt any three of the following (5 MKS each)**

1) Prove that a system of linear equations has either no solution or unique solution or infinitely many solutions.

2) Prove that for  $A, B$  matrices of order  $m \times n$  then  $A, B$  are row equivalent iff there exists a invertible matrix  $P$  such that  $B = PA$

3) Prove that  $A$  is invertible iff  $A$  can be expressed as product of elementary matrices.

4) Solve the following system of linear equations using Gauss elimination method  
 $4x+3y=7, x-4y=6, 10x-2y=15$

**Q.3) Attempt any three of the following (5 MKS each)**

1) Prove that  $P[x]$  denote set of all polynomials with real coefficients then it is a vector space over  $\mathbb{R}$  with respect to usual addition and scalar multiplication of polynomials.

2) Check whether  $A$  is a subspace of  $\mathbb{R}^2$  where

$$A = \{(x, 4+x) / x \in \mathbb{R}\}$$

3) Let  $(V, +, *)$  be a real vector space,  $W_1, W_2$  two subspaces of  $V$  then prove that intersection of  $W_1$  and  $W_2$  is also a subspace of  $V$ .

4) Prove that  $S = \{(1,0), (0,1)\}$  is a basis of  $\mathbb{R}^2$ .

**Q.4) Attempt any three of the following (5 MKS each)**

1) Define a Kernel of a linear transformation and find Kernel of  $T$  for  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$  as

$$T(x, y) = (x+2y, x-2y, 3x+4y)$$

2) Let  $V$  be a finite dimensional real vector space,  $T: V \rightarrow V$  be a linear transformation then  $\text{Ker } T = \{O\}$  iff  $\text{Im } T = V$

3) Prove that a linear transformation  $T: V \rightarrow W$  is invertible iff  $T$  is an isomorphism

4) If  $T: V \rightarrow W$  is one one linear transformation then what should be the relation between  $\dim V$  and  $\dim W$ ?

**Q.5) Attempt any one of the following ( 5 MKS each)**

1) Show that the matrices  $A, B$  are row equivalent by producing a sequence of elementary row

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 1 \\ 2 & 1 & 9 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 0 & 5 \\ 0 & 2 & -2 \\ 1 & 1 & 4 \end{bmatrix}$$

operations that produces  $B$  from  $A$ .

2) Let  $(V, +, *)$  be a real vector space then prove that additive identity is unique.

3) Check whether the following is a linear transformation or not

$T: \mathbb{R} \rightarrow \mathbb{R}$  as  $T(x) = x^2$