VCD 5|10|17 - MATHS I - FYBSC - SEM II ATKT EXAM - 75 MARKS - $2\frac{1}{2}$ HRS

NOTE :
1) All questions are compulsory.
2) For Q.1, Q.2 and Q. 3 attempt any one subquestion (each 8 marks) from part (a), and any three subquestions (each 4 marks) from part (b).
3) For Q.4 , attempt any three.(each 5 marks)

Q.1. (a) Attempt any one. [each 8]

1) Let Σa_n, Σb_n be convergent series converging to a & b respectively then prove following

i) If $c_n = a_n + b_n \quad \forall\, n \epsilon \mathbb{N}$ then Σc_n is convergent, converging to $a + b$.

ii) If $\alpha \,\epsilon\, \mathbb{R}$ is fixed and $c_n = \alpha\, a_n$ then Σc_n is convergent, converging to αa.

2) Examine convergence of following series.

i) $\sum \frac{n}{n^2 - cos^2(n)}$ ii) $\sum \frac{n^2+2}{n^4+5}$

(b) Attempt any three. [each 4]

1) Find n^{th} partial sum and determine if the series converge or diverge.

$$1 - 3 + 9 - 27 + \cdots + (-1)^{n-1}(-3)^{n-1}$$

2) Define an absolute convergence of series and prove that absolute convergent series are convergent.

3) State Leibnitz theorem and examine the convergence of $\frac{(-1)^{n+1}}{n}$

4) State Modified root test and examine the convergence of $\sum_{n=1}^{\infty} \frac{4^n}{n^2}$

Q.2. (a) Attempt any one. [each 8]

1) Define the derivative of a real valued function f at $x \,\epsilon\, I$ an open interval subset of $\mathbb{R}$ and show that f is everywhere differentiable on $\mathbb{R}$ where $f: \mathbb{R} \to \mathbb{R}$ as $f(x) = x^n \quad n \epsilon \mathbb{N}$

2) Differentiate following function with respect to x using rules of differentiation.

i) $\sqrt{\log x}$ ii) $\frac{x^n}{\log x}$ iii) $e^x(5 - \sqrt{x})$

(b) Attempt any three. [each 4]

1) Find equation of tangent and normal to the following

$x^2 + xy + 2y^2 = 28$ at $(2,3)$

2) If $y = x + \tan x$ then prove that $cos^2 x\, y_2 + 2(x - y) = 0$

3) If $-1 \le x \le 1$ then find derivative of $y = \cos^{-1} x$ $\quad 0 < y < \pi$

4) When a real valued function $f: I \to \mathbb{R}, I \subseteq \mathbb{R}$ open interval, said to be a differentiable on I? And check the differentiability

Q.3. (a) Attempt any one. [each 8]

1) State Rolle's Theorem and give their geometrical interpretation.

Verify Rolle's Theorem for $f(x) = (x^2 - 1)(x - 2)$ in $[-1,2]$

2) State Lagranges Mean Value Theorem and give their geometrical interpretation.

Verify Lagranges Mean Value Theorem for $f(x) = \sqrt{x^2 - 4}$ in $[2,3]$

(b) Attempt any three. [each 4]

1) Expand $x^5 - x^4 + x^3 - x^2 + x - 1$ in powers of $(x - 1)$ and find $f(0.99)$

2) State L-Hospital's Rule and evaluate following

$\lim_{x\to 1} \frac{1+\log x - x}{1-2x+x^2}$

3) Find absolute maximum and minimum values of each function on the given interval. $f(x) = \frac{-1}{x^2}$ $\quad [0.5,2]$

4) Divide the number 22 into two parts so that the sum of their square is minimum

Q.4. Attempt any three. [each 5]

1) Prove that the series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges if $p > 1$ and diverges if $p \le 1$

2) Define the following term

i) An infinite series

ii) n^{th} partial sum of the series

iii) Geometric series

iv) Harmonic series

v) An alternating series

3) Is $f(x) = |x|$ differentiable on $\mathbb{R}$?

4) If a function $f: I \rightarrow \mathbb{R}, I \subseteq \mathbb{R}$ open interval, is differentiable at $c \in I$ then prove that f is continuous at $c \in I$. What about converse?

5) Verify Cauchy Mean Value Theorem for $f(x) = \sqrt{x}, g(x) = \frac{1}{\sqrt{x}}$ in $[a, b]$

6) Sketch the graph of $y = x^3 - 3x^2 + 4$