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-ATKT
2015-16

VCD - 30/3/15 - MATHS II - FYBSC - SEM II EXAM - 75 MARKS - 2.5HRS -220

Note: : 1) All questions are compulsory.

2)For Q.1, Q.2, Q.3, attempt any one subquestion (each 8 mks)from part (a), and any three subquestions (each 4 mks) from part(b)

3) For Q.4 Attempt any three.(each 5 mks)

Q.1 (a) Attempt any one [Each 8]

1) Solve following equations using Gaussian elimination method

$$x + 7y + 3z = 11$$

$$x + y + z = 3$$

$$4x + 10y - z = 13$$

2) Write the note on elementary transformation and define following with example.

i) Transpose of matrix

ii) Diagonal matrix

iii) Symmetric matrix

iv) Upper and lower triangular matrix

Q.1 (b) Attempt any three. [Each 4]

1) Find Parametric equation of a plane passing through points (1,2,3), (4,5,6), (7,8,9)

2) Give geometric interpretation of solution of system of m homogeneous linear equations in n unkowns.

3) Define an invertible matrix and prove that $(A^{-1})^T = (A^T)^{-1}$ where A^{-1}, A^T are inverse and transpose of A respectively.

4) Define addition and multiplication of matrices and find AB, BA for following

$$A = \begin{bmatrix} 3 & 2 \\ 1 & 5 \\ 2 & 7 \end{bmatrix}, B = \begin{bmatrix} 4 & 2 & 3 \\ 7 & 1 & 5 \end{bmatrix}$$

Q.2 (a) Attempt any one [Each 8]

1) Prove that $(\mathbb{R}[x], +, \circ)$ is a vector space over $\mathbb{R}$ where $\mathbb{R}[x]$ = Set of all polynomials in x with real coefficients.

(PTO)

2) Let V be a vector space over $\mathbb{R}$ and W be a nonempty subset of V. Then prove that W is a subspace of V iff $ax + by \in W$ whenever $x, y \in W,\ a, b \in \mathbb{R}$

Q.2 (b) Attempt any three. [Each 4]

1) Show that intersection of finitely many subspaces of a vector space V is also a subspace of V.

2) Express given vector $x = (1,2,0) \in \mathbb{R}^3$ as linear combination of given vectors $x_1 = (1,0,0), x_2 = (0,1,1), x_3 = (0,1,3)$

3) Find Linear Span of $S = \{(1,1),(0,2)\}$ of $\mathbb{R}^2$

4) Define a subspace W of a vector space V and prove that

$W = \{(x, y, z) : z = x + y,\ x, y, z \in \mathbb{R}\}$ a subset of $\mathbb{R}^3$ is a subspace of $\mathbb{R}^3$.

Q.3 (a) Attempt any one [Each 8]

1) Prove that rotation of a vector X through an angle θ in anticlockwise direction is a linear transformation.

2) State Rank –Nulity Theorem and verify it for following

$T: \mathbb{R}^2 \to \mathbb{R}^3$ defined by $T(x, y) = (x, x + 2y, y + 3x)$

Q.3 (b) Attempt any three. [Each 4]

1) When is the map $T: V \to U$ where V, U are vector spaces over $\mathbb{R}$ said to be a linear transformation? check whether following map is a linear transformation or not.

$T: \mathbb{R}^2 \to \mathbb{R}^2$ defined by $T(x, y) = (y, 0)$

2) Let V, U be a vector spaces over $\mathbb{R}$. If a transformation $T: V \to U$ is such that $T(ax + by) = aT(x) + bT(y)\quad \forall x, y \in V,\ a, b \in \mathbb{R}$ then prove that T is a linear transformation.

3) Let V, V' be a vector spaces over $\mathbb{R}$ and $T: V \to V'$ be a linear transformation then prove following

i) Ker T is a subspace of V

ii) Img T is a subspace of V'.

4) Let V, U be vector spaces over $\mathbb{R}$. If $T: V \to U$ is a linear transformation then prove that

i) $T(0) = 0$ ii) $T(-x) = -T(x)\quad \forall x \in V$

Q.4 (a) Attempt any three [Each 5]

1)Define the row echelon form of a matrix and reduce following matrix in row echelon form.

$$A = \begin{bmatrix} 3 & 3 & 9 \\ 0 & 5 & 0 \\ 5 & 8 & 1 \end{bmatrix}$$

2) Find inverse of $A = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 1 & 1 \\ -1 & -2 & -3 \end{bmatrix}$ using elementary row transformation.

3) If $a \neq 0, a \in \mathbb{R}$ then show that $\{(1 + a, 1 - a), (1 - a, 1 + a)\}$ is linearly independent subset of $\mathbb{R}^2$

4) Express the function $1 + 2t + 3t^2$ as linear combination of $1, 1 + t, 1 + t^2$

5) Prove that every finitely generated vector space has a finite basis.

6) Prove that Sum of Linear Transformation is also a Linear Transformation.