VCD/ 011222 FYBSC- SEM I MATHEMATICS – II 75 MARKS $2\frac{1}{2}$ HRS (240)

Note: 1. All questions are compulsory.
2. Figures to the right indicate marks.
3. Draw neat, labelled diagrams wherever necessary.

1. a) Attempt any **ONE** question from the following: **(08)**
   i. State and prove Binomial theorem for $n \in \mathbb{N}$
   ii. Prove, for every integer $n > 1$ can be expressed as a product of positive primes and this expression is unique for order in which prime factors occur.

   b) Attempt any **TWO** questions from the following: **(12)**
   i. Prove that by using finite method of induction $3^{2n+2} - 8n - 9$ is divisible by 64 for $n \in \mathbb{N}$
   ii. Find the smallest positive integer to which 10^{515} is congruent modulo 7.
   iii. Prove that $((a, b), c)) = (a, (b, c))$

2. a) Attempt any **ONE** question from the following: **(08)**
   i. Let a, b ∈ $\mathbb{Z}$ and n ∈ N. Define a relation R in $\mathbb{Z}$ as,
   aRb iff a ≡ b (mod n) then prove that R is an equivalence relation.
   ii. Define: Binary operation, commutativity, associativity, existence of identity element and existence of inverse element. Check all the properties for
   $a * b = |ab|$ in $\mathbb{R} \setminus \{0\}$

   b) Attempt any **TWO** questions from the following: **(12)**
   i. Write the distinct residue classes modulo 6 and the addition table modulo 6.
   ii. Define partition of a set and list any 5 partitions of set {a, b, c, d, e}
   iii. Check whether $f: \mathbb{R} \setminus \left\{\frac{-2}{7}\right\} \to \mathbb{R} \setminus \left\{\frac{2}{7}\right\}$ given by $f(x) = \frac{2x-3}{7x+2}$ is bijective.

3. a) Attempt any **ONE** question from the following: **(08)**
   i. Define Divisibility in $\mathbb{R}[x]$. State Division Algorithm in $\mathbb{R}[x]$. Also state and prove Remainder Theorem.
   ii. State and prove Unique Factorization Theorem in $\mathbb{R}[x]$.

   b) Attempt any **TWO** questions from the following: **(12)**
   i. Find the G.C.D. of polynomials $f(x) = x^8 - 1$ and $g(x) = x^{12} - 1$ over $\mathbb{Q}[x]$.
   ii. Define irreducible polynomials. And if p(x) is irreducible polynomial in $\mathbb{R}[x]$. Prove that, If p(x) does not divide a(x) in $\mathbb{R}[x]$, then (p(x), a(x)) = 1.
   iii. Find the multiplicity of each root of $f(x) = 4x^3 + 4x^2 - x - 1$

4. Attempt any **THREE** questions from the following: **(15)**
   a) Show that 3927 and -377 are co-prime.
   b) Define Euler ϕ- function and hence find $\phi(5040)$
   c) Check whether following relation is reflexive, symmetric, transitive or equivalence relation: xRy if $17 \mid x - y$ in $\mathbb{R}$
   d) Check whether the function $f: \mathbb{Q} \to \mathbb{R}$ given by $f(x) = 2x + 3$ is bijective or not.
   e) If r_1, r_2, r_3 are the roots of polynomial $x^3 - 4x^2 + 5x + 1$, without actually calculating the values of r_1, r_2, r_3, write polynomial with roots $3r_1$, $3r_2$, and $3r_3$.
   f) Find the cube roots of unity.

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