22/11/19

VCD -- - MATHS I- FYBSC - SEM I EXAM - 100 MARKS - 3HRS

N.B : 1) All questions are compulsory

2) Figures to right indicate full marks

Q.1 Choose correct alternative in each of the following (2 marks each)

1) If $0 < x < y$ *and* $0 < z < w$ *then* $----$

a) $0 < xw < yz$ b) $0 < yz < xw$ c) $0 < xz < yw$ d) none of these

2) Infimum of the set $\left\{1, \frac{1}{2}, \frac{1}{3}, \ldots\ldots\right\}$ is -------

a) 1 ab) 0 c) $\frac{1}{10}$ d) none of these

3) The sequence $x_n = (-1)^n n$ *is* $----$

a) convergent b) divergent c) oscillates infinitely d) none of these

4) If $(x_n) = \sin n\Pi$ $n \in \mathbb{N}$ *then* (x_n)*is* $-----$

a) bounded and convergent b) bounded but not convergent

c) unbounded but converngent d) none of these

5) A monotonic sequence ------

a) must be bounded b) may be bounded c) can not be bounded d) none of these

6) The sequence $(1 + \frac{1}{n})^n$ is --------

a) monotonic and bounded b) neither monotonic nor bounded

c) monotonic but not bounded d) none of these

7) A convergent sequence of rational numbers ------

a) has a rational limit b) may have irrational limit

c) cannot have irrational limit d) none of these

8) Which of the following is true?

a) A sequence is Cauchy if it is bounded b) a sequence is convergent if it is bounded

c) a sequence is convergent if it is cauchy d) none of these

9) $\lim_{x \to 0} x^2 \sin\frac{1}{x}$ is-------

a) 0 b) 1 c) -1 d) does not exist

10) $f(x) = \begin{cases} \sin\frac{1}{x}, & x \neq 0, x \in \mathbb{R} \\ 0, & x = 0 \end{cases}$

a) f is continuous at all points in $\mathbb{R}$ c) f is discontinuous at all points in $\mathbb{R}$

b) f is continuous at all points other than zero in $\mathbb{R}$ d) none of these

Q.2 a) Attempt any ONE question from the following.(8marks each)

1) State and prove Arithmetic Mean and geometric mean inequality for two nonnegative real numbers a,b. Using it prove that if $a, b, c \in \mathbb{R}^+$ $(a+b)(a+c)(b+c) \geq 8abc$

2) State and prove Cauchy Schwarz inequality of $\mathbb{R}$ and using it prove that if a, b *are real numbers then prove that* $3(a^2+b^2+c^2) \geq (a+b+c)^2$.

b) Attempt any TWO question from the following.(6marks each)

1) State and prove Hausdorff Property of $\mathbb{R}$ and by applying it find disjoint neighbourhood of $a = \frac{2}{3}, b = \frac{3}{2}$.

2) Define the following term

Upper bound , Lower bound, least upper bound ,greatest lower bound and prove that For a nonempty subset S of $\mathbb{R}$, Supremum of S whenever exists is unique.

3) Prove that a nonempty subset of $\mathbb{R}$ which is bounded below has the infimum in $\mathbb{R}$.

Also find infimum and supremum of the following set $\left\{-1, \frac{-1}{2}, \frac{-1}{3}, \ldots\ldots\right\}$

4) Let S be a nonempty subset of $\mathbb{R}$. *Prove that* a real number m is the infimum of S iff
i) $x \geq m \quad \forall x \in S$ ii) $\forall \epsilon > 0 \ \exists\, x \in S$ *such that* $x < m + \epsilon$

Q.3 a) Attempt any ONE question from the following.(8marks each)

1) i) If $(x_n) \to x$ *and* $\alpha \in \mathbb{R}$, then prove that $(\alpha x_n) \to \alpha x$.

ii) If $x_n \neq 0 \ n \in \mathbb{N}, x \neq 0$ then prove that $(x_n) \to x \Rightarrow \left(\frac{1}{x_n}\right) \to \left(\frac{1}{x}\right)$

2) State and prove Sandwich theorem for sequence. Hence discuss the

convergence of $i)\ a_n = \frac{\sin n}{n}$, $ii)\ b_n = \frac{1}{n!}$

b) Attempt any TWO question from the following.(6marks each)

1) i) Examine whether the following sequences are monotonic? $i)\ a_n = (-1)^n$ $ii)\ a_n = \sin n$

ii) Prove that every monotonic increasing sequences converges to its lub if bounded above.

2)i) Define a subsequence of a sequence x_n of $\mathbb{R}$ and prove that If (x_{n_k}) is a subsequences of x_n Converging to p in $\mathbb{R}$ then prove that (x_{n_k}) converges to p.

ii) Find the two convergent subsequences of (x_n) and discuss the convergence of (x_n) if

$$(x_n) = \cos n\pi + \frac{2}{n}$$

3) A sequence (a_n) is given by $a_1 = \sqrt{2}$ and $a_{n+1} = \sqrt{2a_n}$ then show that (a_n) converges to 2

4)i) Show that the following sequences are convergent and find where the limit of each lies

$a_n = \frac{1}{3+1} + \frac{1}{3^2+1} + \frac{1}{3^3+1} + \cdots \ldots. + \frac{1}{3^n+1}$

ii) prove that every convergent sequence in $\mathbb{R}$ is a Cauchy sequence.

Q.4 a) Attempt any ONE question from the following.(8marks each)

1) i) If $\lim_{x\to a} f(x)$ exists then prove that it is unique

ii) Let $\lim_{x\to a} f(x)$ *exists at* $a \in \mathbb{R}$ *then prove for* k *non zero real number,*

$$\lim_{x\to a} k(f(x)) = k \lim_{x\to a} f(x)$$

2) State and prove the sequential continuity theorem for real valued continuous function.

b) Attempt any TWO question from the following.(6marks each)

1)) Let $\lim_{x\to a} f(x)$ & $\lim_{x\to a} g(x)$ *exist at* $a \in \mathbb{R}$ *then prove that*

$$\lim_{x\to a} f(x)\, g(x) = \lim_{x\to a} f(x) . \lim_{x\to a} g(x)$$

2) State and prove Sandwich theorem for real valued functions.

3) Using $\varepsilon - \delta$ definition show that i) $\lim_{x\to 2} 2x + 5 = 9$ ii) $\lim_{x\to 2} 4 = 4$

4) Discuss the continuity of following function

i) $f(x) = \begin{cases} \frac{1}{x-1}, & x \neq 1 \\ 0 & x = 1 \end{cases}$ ii) $f(x) = \begin{cases} x, & x \in \mathbb{Q} \\ 0, & x \in \mathbb{R} - \mathbb{Q} \end{cases}$

Q.5 Attempt any FOUR question from the following.(5marks each)

1) Define the convergence of a sequence of $(x_n)$of $\mathbb{R}$ and Prove that *the sequence* $\left(\frac{1}{n}\right)$ converges to zero.

2) Prove that every convergent sequence is bounded.

3) Discuss $\lim_{n\to\infty} a^n$ *for the given cases*

$case\ i)\ a \geq 1, \qquad case\ ii) -1 < a < 1$

4) Using $\varepsilon - n_0$ definition show that $\lim_{n\to\infty} \frac{5n}{n+1} = 5$

5) Find the domain and the range of each function

i) $i)\ f(x) = x^2 + 4 \qquad ii)\ f(x) = \frac{1}{1+\sqrt{x}}$

Also draw the graph of given function $f(x) = \begin{cases} \sin x, & -\pi \leq x \leq 0 \\ 1, & 0 < x \leq \pi \end{cases}$

6) If $2x \leq g(x) \leq 2\cos(x-1) \quad \forall x \in \mathbb{R} \qquad then\ find\ \lim_{x\to 1} g(x)$