

- Note : i) All the questions are compulsory.
ii) Figures to the right indicate marks.
iii) Illustrations in depth answers and diagrams will be appreciated.
iv) Mixing of sub – question is not allowed.

Q. 1. Attempt all (each of 5 marks)

(15)

a) Choose the best choice for the following questions :

i) Which of the following are defined if f is a function from A to B and g is a function from B to C

- a) $f \circ g$ but not $g \circ f$ b) both $f \circ g$ and $g \circ f$
c) $g \circ f$ but not $f \circ g$ d) neither $f \circ g$ nor $g \circ f$

ii) Let $\{a_n\}$ be a sequence such that $a_n = a_{n-1} + 3$ for all $n \geq 1$ & suppose $a_0 = 2$. What are a_1 & a_2

- a) 5 and 8 respectively b) 3 and 5 respectively
c) 8 and 5 respectively d) 5 and 3 respectively.

iii) The value of $P(10, 2)$ is

- a) 180 b) 45 c) 90 d) none of these

iv) Which of the following types of grammar has no restrictions on its productions?

- a) Type 0 b) Type 1 c) Type 2 d) Type 3

v) A graph with multiple edges but no loose is called a

- a) simple graph b) compound graph
c) Compound graph d) multigraph

b) Fill in the blanks . Use following pool to answer the questions.

(05)

Pool (complete , equivalence , injective , regular , 6,12,67,600, partial order , surjective)

i) A one one function is called as _____ function.

ii) Coefficient of $x^2 y^2$ in the expansion of $(x + y)^4$ is _____

iii) A graph in which degree of every vertex is same is called _____ graph.

iv) A relation which is reflexive , antisymmetric & transitive is called _____ relation.

v) _____ is the number of different license plates that can be formed if each plate contains a sequence of 2 capital letter followed by sequence of 2 digits.

c) Define the following:

(05)

1. Pendant vertex
2. Binomial theorem

3. Partial order relation
4. Inclusion exclusion principal
5. Regular grammar

Q 2. Attempt the following (any 3)

(15)

- a) Consider the relation $R = \{(1, 1), (1, 2), (2, 1), (2, 3), (3, 4), (3, 1)\}$ on $A = \{1, 2, 3, 4\}$. Draw its directed graph and find matrix M_R of R .
- b) Show that a function $f: R \rightarrow R$ defined by $f(x) = 2 - 3x$ is bijective. Hence, find f^{-1} .
- c) Solve the recurrence relation $a_n = a_{n-1} + 2a_{n-2}$ for all $n \geq 2$, whether $a_0 = 2$ & $a_1 = 7$.
- d) Determine whether the relation of $R = \{(1, 1), (1, 2), (2, 1), (3, 3)\}$ on set $A = \{1, 2, 3\}$ is reflexive, symmetric and transitive. Justify
- e) Define composition of 2 functions. If f and g are two functions from set of integers to set of integers defined by $f(x) = 3x - 5$ and $g(x) = 2x^2$, then find $f \circ g(x)$ and $g \circ f(x)$
- f) Describe Tower of Hanoi puzzle. Formulate and solve the recurrence relation for it.

Q 3. Attempt the following (any 3)

(15)

- a) State and prove Pascal's identity.
- b) How many words can be formed using all the letter of the word MATHEMATICS?
- c) State pigeonhole principal. Show that if any six numbers from the set $\{1, 2, 3, 4, \dots, 10\}$ are chosen, Then two of them will add upto 11.
- d) Among 100 students, 55 students got distinction in first year, 30 got distinction in second year, 15 got distinction in both years. Then, how many students got distinction in
 1. at least one year
 2. Only first year
 3. only second year.
- e) Construct a derivation tree for following sentence 'A scared mouse runs quickly'
- f) Let G be a grammar where $V = \{s, o, 1\}$, $T = \{0, 1\}$, starting symbol S and set of production $P = \{S \rightarrow 11S, S \rightarrow 0\}$. Find $L(G)$.

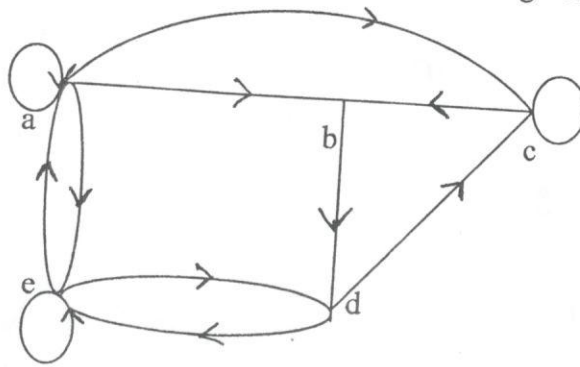
Q 4. Attempt the following (any 3)

(15)

- a) Define adjacency matrix. Draw the undirected graph G corresponding to given adjacency matrix.

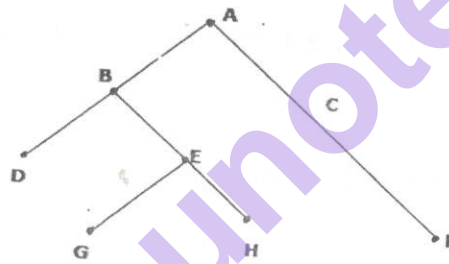
$$A = \begin{bmatrix} 0 & 2 & 1 & 1 \\ 2 & 0 & 2 & 1 \\ 1 & 2 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix}$$

- b) Find in degree and out degree of each vertex in the given graph



- c) What is a complete graph? Draw a complete graph with 5 vertices. Give an example of a graph which is regular but not complete.

- d) Consider the binary tree T in the following figure. Traverse T in a) preorder b) inorder c) postorder

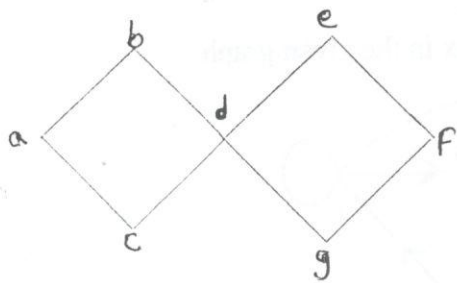


- e) Represent the following expressions using binary tree

1) $(x + xy) + (x / y)$

2) $x \pm ((xy + x) / y)$

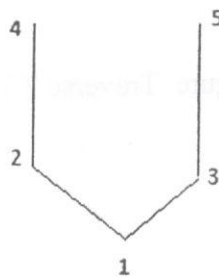
- f) Use depth first search algorithm to find a spanning tree for the given graph.



Q 5. Attempt the following (any 3)

(15)

- a) Describe the ordered pairs in the relation R determined by the Hasse diagram of poset (A, \leq) on the set $A = \{1, 2, 3, 4, 5\}$



- b) Solve the recurrence relation $a_n = 3 a_{n-1}$ for all $n \geq 1$ with initial condition $a_0 = 2$, using generating function method.
- c) Draw all possible non similar binary tree T with 3 nodes
- d) There are 6 men and 7 women in a group. How many different committee should have 2 men and 3 women ?
- e) Let M be the finite state machine with state table as given below

F	X	y
S_0	S_1, a	S_2, b
S_1	S_2, a	S_1, C
S_2	S_0, C	S_1, b

- 1) Find input set A , the state set S , the output set Z , the initial state.
- 2) Draw the state diagram of M .