Y CD 23/11/19 FYBSC(PCM) SEM – I SUBJECT :- ALGEBRA

100 MARKS, $2^{1/2}$ HOURS

NOTE : - 1) All questions are compulsory.

2) Figures to right indicate full marks of each sub-question.

Q.1) Choose correct alternative in each of the following. (20)

I. a) Every subset of positive integers has the least element.
b) Every non-empty subset of positive integers has the largest element.
c) Every non- empty subset of N, has the least element.
d) 1 is the least element of every non-empty subset of positive integers.

II. For integers r, s and n, n > o the value of $(r+s)^n + (r-s)^n$?
a) Is always even. b) Is always odd.
c) Depends on n. d) Depends on r & s.

III. If a/b and a/c then a^2/ bc is?
a) Always true. b) Never true
c) Valid only when a = o d) Valid only when a = 1.

IV. The integers 15239 and 15240 are?
a) Co-prime b) Not co-prime
c) Their g.c.d. is O d) Their g.c.d. is a prime.

V. If every pair of integers satisfy a = b (modn) then n is?
a) 0 b) 1 c) 2 d) 3

VI. The unit digit of 3^{10} is…?
a) 0 b) 9 c) 1 d) -1

VII. A function from A to B, is b is bijective, if it is…?
a) One-one b) Onto c) One-one and onto d) Depends on A and B

VIII. A relation on ℤ, given by a and b if $a \neq b$ is…?
a) Reflexive b) Symmetric c) Transitive d) An equivalence relation

IX. A monic polynomial is the one whose…?
a) Degree is 1 b) Leading coefficient is 1 c) Constant term is 1 d) All the coefficients are 1

X. If w is a complex n^{th} root of $x^n = 1$ such that, $1 + w + w^2 + w^3 + w^4 = 0$ then the smallest positive value of n is….?
a) 4 b) 5 c) 6 d) 9

Q.2 A) Attempt only ONE question from the following. (08)

I. State and prove division algorithm in $\mathbb{Z}$.

II. Prove that every integer $n > 1$, can be expressed as a product of positive primes also prove that expression is unique except for the order in which the prime factors occur.

Q.2 B) Attempt any TWO questions from the following. (12)

I. Using method of finite induction prove that the following example is true for all $n \in \mathbb{N}$
$1\cdot2+2\cdot3+3\cdot4+\ldots..n(n+1)=\frac{n}{3}(n+1)(n+2)$

II. Prove in usual notations that for $n \in \mathbb{N}$.

a) ${}^nc_0 + {}^nc_1 + {}^nc_2 + \ldots\ldots {}^nc_n = 2^n$

b) ${}^nc_0 - {}^nc_1 + {}^nc_2 + \ldots\ldots (-1)^n\ {}^nc_n = 0$

III. State and prove Euclid's Lemma

IV. If m and n are two integers such that $(m, n) = 1$ then prove that $\emptyset(mn) = \emptyset(m)\emptyset(n)$

Q.3 A) Attempt any ONE question from the following. (08)

I. F : X→ Y and A & B are non-empty subsets of X and Y respectively. Then prove that.

a) $A \subseteq f^{-1}(f(A))$ then equality holds if and only if f is injective.

b) $f(f^{-1}(b)) \subseteq B$ The equality holds if and only if f is surjective.

II. If ~ is equivalence relation on a non-empty set X, then prove the following.

a) Each element of X belongs to some equivalence class of X.

b) Any two equivalence classes of X are either disjoint or identical.

c) Union of these equivalence classes is X.

Q.3 B) Attempt any TWO questions from the following. (12)

I. $f: \mathbb{R} - \{-\frac{3}{5}\} \to \mathbb{R} - \{\frac{9}{5}\}$, given by $\frac{9x+5}{5x+3}$
prove the above function with the given domain and co-domain is a bijection. Also find corresponding inverse function.

II. F : x→ y is a function B_1, B_2 are any two non-empty subsets of y, prove that.
$B_1 \subseteq B_2 \Longrightarrow f^{-1}(B_1) \subseteq f^{-1}(B_2)$ Is the converse true? Justify your answer.

III. A binary operation 'o' is defined on $\varphi - \{0\}$ as follows $aob = \frac{ab}{6}$ for a, b in $\varphi - \{0\}$
Show that 'o' satisfies all the properties stated in binary operation.

IV. For the following relation, defined on the given set x, find whether it is (i) reflexive (ii) symmetric (iii) transitive if the relation is equivalence state its equivalence classes.
X : the set of integers $\mathbb{Z}$
R : for a, b ∈ $\mathbb{Z}$, $a R b$ if and only if 2a + b is divisible by 3.

Q.4 A) Attempt any ONE question from the following. (08)

I. Prove that a polynomial in C[x] of degree n has exactly n complex roots, counted with multiplicities.

II. State and prove rational root theorem.

Q.4 B) Attempt any TWO questions from the following. (12)

I. State and prove Remainder Theorem.

II. By dividing f(x) by g(x) find the quotient and remainder in $\mathbb{R}[x]$
$F(x) = x^3 - 4x^2 + x + 6, \quad g(x) = x^2 - 1$

III. If p(x) | a(x) b(x), a(x), b(x) in $\mathbb{R}[x]$ then prove that p(x)/ a(x) or p(x)/ b(x) in $\mathbb{R}[x]$

IV. Find all rational roots and their multiplicity of $12x^3 - 16x^2 + 7x - 1$

Q.5) Attempt any FOUR questions from the following. (20)

I. Find the gcd of the given pair of numbers and express it as a linear combination of the given pair 1008, 357.

II. Verify Wilson's Theorem for p = 13.

III. Define the following terms.

a) Cartesian product of two sets.

b) Functions [mappings]

IV. $F : X \to Y, g : Y \to Z$ are two functions such that, gof is bijective and f is subjective, then prove that, g is injective.

V Let $f(x) \in \mathbb{R}[x]$ let $\propto \in C$, be a root of f(x) = 0 then prove that conjugate of $\propto$ is also a root of f(x) = 0

VI. Find all roots of $f(x) = x^3 - 3x^2 - 4x - 12$ if sum of its two roots is zero.

****