VCD - 10/10/17 - MATHS II - FYBSC - SEM I ATKT EXAM - 75 MARKS - $2\frac{1}{2}$HRS

NOTE : 1) All questions are compulsory.

2) For Q.1, Q.2 and Q. 3 attempt any one subquestion (each 8 marks) from part (a), and any three subquestions (each 4 marks) from part (b).

3) For Q.4 , attempt any three.(each 5 marks)

Q.1. (a) Attempt any one. [each 8]

1) Define Euler φ function $\varphi(n)$ for positive integer $n \geq 1$ and prove that If $m, n \in \mathbb{Z}$ such that $(m, n) = 1$ then

$$\varphi(m, n) = \varphi(m)\,\varphi(n) \text{ and also find } \varphi(210).$$

2) Find greatest common devisor of $2210, 357$ and express it in form of $2210m + 357n$ $m, n \in \varphi$. Are m, n unique? Justify

(b) Attempt any three. [each 4]

1) State First principle of finite induction and prove that $n(n+1)(n+2)$ is divisible by 6 $\forall n \in IN$.

2) Using Pascal's triangle, expand $(a+b)^8$.

3) Prove that The number of primes are infinite.

4) Define greatest common devisor of non zero integer a & b and prove that the positive gcd of any two integers (whenever exists) is unique.

Q.2. (a) Attempt any one. [each 8]

1) Define invertible function and prove that let $f : A \to B$ & $g : B \to C$ be invertible then gof is invertible & $(gof)^{-1} = f^{-1}og^{-1}$

2) Define i) Equivalence relation R on non empty set X

ii) partition of non empty set X.

and prove that if P is partition of non empty set X then P induces equivalence relation on X.

(b) Attempt any three. [each 4]

Check whether $*$ is binary on given set

i) $a * b = 5a - b$ on $\mathbb{Z}^+$

ii) $a + b = a \div b$ on $\mathrm{IR} - \{0\}$

2) Determine whether each relation from A to B. If it is function give its range.

$A = \{a, b, c, d\}$, $B = \{1,2,3,4\}$

$g = [(a, 1), (b, 1), (c, 1), (d, 1)]$, $h = \{(a, 2), (c, 3), (d, 3), (b, 4)\}$

3) Let $f : IR - \{3\} \rightarrow IR - \{0\}$ be defined by $f(x) = \frac{1}{x-3}$ then prove that f is bijective and Find formula for f^{-1}.

4) Determine whether following relation R on set A is equivalence or not

$a\ R\ b$ iff $a + b$ is even $\quad a, b \in \mathbb{Z} = A$.

Q.3. (a) Attempt any one. [each 8]

1) State and prove Remainder theorem for polynomial $f(x) \in F[x]$ and compute the remainder when $f(x)$ is divided by $g(x)$.

$f(x) = x^4 - 3x^2 + 4x + 8$

$g(x) = x + 2$

2) State and prove Factor theorem. Use it to determine whethe or not $g(x)$ is factor of $f(x)$.

$f(x) = x^4 + 4x^3 + 6x^2 + 4x + 1$, $g(x) = x + 1$

(b) Attempt any three. [each 4]

1) Prove that a non constant polynomial $f(x) \in F[x]$ can be expressed as product of linear and quadradic polynomial.

2) Express $5i$ in polar form and also find magnitude and amplitude.

3) Use De Moivre's theorem to prove that

$\sin 3\theta = 3cos^2\theta \sin\theta - sin^3\theta$

$\cos 3\theta = cos^3\theta - 3\cos\theta\, sin^2\theta$

4) Find quotient and remainder when $f(x)$ is divided by $g(x)$

$f(x) = x^3 - 2x^2 + 3x - 7$

$g(x) = x^2 + 2$

Q.4. Attempt any three. [each 5]

1) State and prove Euclid's Lemma.

2) Using Euler's theorem prove $5^{303} = 4(mod\ 11)$.

3) If R is an Equivalence relation on a non empty set X then prove that any two equivalence classes of X are either identical or disjoint.

4) Show that $f: A \rightarrow B$, $g: B \rightarrow C$ are function then for any nonempty subset X of A.

$$(gof)(X) = g(f(X))$$

5) State and prove Rational Root theorm.

6) Use De Moivre's theorem to prove that

$$\sin 3\theta = 3cos^2\theta \sin\theta - sin^3\theta$$

$$\cos 3\theta = cos^3\theta - 3\cos\theta \, sin^2\theta$$