

NOTE : 1) All questions are compulsory.

- 2) For Q.1, Q.2 and Q. 3 attempt any one subquestion (each 8 marks) from part (a), and any three subquestions (each 4 marks) from part (b).
3) For Q.4 , attempt any three.(each 5 marks)

Q.1. (a) Attempt any one. [each 8]

1) For $a, b, c \in \mathbb{R}$ prove the following

- i) $a < b \Rightarrow -a > -b$
- ii) $a < b \Rightarrow a + c < b + c$
- iii) $a > b \text{ \& } c < 0 \Rightarrow a < c$
- iv) $a > b \text{ \& } c > 0 \Rightarrow ac < bc$

2) State all properties of addition of $\mathbb{R}$ = set of real numbers.

(b) Attempt any three. [each 4]

1) Find LUB and GLB of $S = \{1/n / n \in \mathbb{N}\}$

LUB= Least Upper Bound

GLB = Greatest Lower Bound

- 2) State and prove AM – GM inequality of $\mathbb{R}$.
- 3) State and prove Hausdorff property of neighbourhood.
- 4) Prove that if a_n is convergent sequence then it is bounded.

Q.2. (a) Attempt any one. [each 8]

1) State and prove Sandwich theorem for sequence. Hence discuss the

convergence of $a_n = \frac{1}{n!}$

2) Define the Cauchy sequence in $\mathbb{R}$ and show that following

sequences are not Cauchy in $\mathbb{R}$

$$i) a_n = (n^2) \quad n \in \mathbb{N} \quad ii) a_n = 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n} \quad \forall n \in \mathbb{N}$$

(b) Attempt any three. [each 4]

- 1) Examine whether the following sequences are monotonic
 - i) $a_n = (-1)^n$
 - ii) $a_n = \sin n$
- 2) Prove that if a_n & b_n are two convergent sequences having limits a and b respectively then prove that $(a_n + b_n)$ converges and $\lim_{n \rightarrow \infty} (a_n + b_n) = a + b$.
- 3) State and prove Cauchy completeness of $\mathbb{R}$.
- 4) Define a subsequence and prove that every bounded sequence in $\mathbb{R}$ has convergent subsequence.

Q.3. (a) Attempt any one. [each 8]

- 1) Let f, g be real valued function defined on subset $J \subseteq \mathbb{R}$ and

$$\lim_{x \rightarrow a} f(x) = l, \lim_{x \rightarrow a} g(x) = m \text{ then prove that } \lim_{x \rightarrow a} f(x) \cdot g(x) = l \cdot m.$$
- 2) prove that $\lim_{x \rightarrow a} f(x) = l$ iff $\lim_{n \rightarrow \infty} f(x_n) = l$ for every sequence x_n converging to a ($x_n \neq a$)

(b) Attempt any three. [each 4]

- 1) Define the following functions
 - i) Constant function
 - ii) Absolute value function
- 2) Prove that Let f be real valued function on subset I of $\mathbb{R}$. Let $P \in I$, if $\lim_{x \rightarrow P} f(x)$ exists then it is unique.
- 3) Define right hand and left hand limits and evaluate right and left hand limits f as $\lim_{x \rightarrow P} f(x)$ in following case

$$f(x) = x - 1 \quad \text{for } x \leq 0$$

$$= 1 - x \quad \text{for } x > 0 \quad \text{at } P = 0$$
- 4) Let $f, g : J \rightarrow \mathbb{R}$ be continuous at $P \in J$ where J is an open interval in $\mathbb{R}$ then prove that $f+g : J \rightarrow \mathbb{R}$ continuous at P .

Q.4. Attempt any three. [each 5]

- 1) Define neighbourhood in $\mathbb{R}$ and prove that intersection of any two neighbourhood $P \in \mathbb{R}$ is the neighbourhood of P .

- 2) Discuss the boundedness of $S = \{x/3 < x \leq 7\}$, $S = \{\sin x/x \in \mathbb{R}\}$
- 3) Show that every monotonic decreasing sequence is bounded above.
- 4) State all algebraic operations on sequences.
- 5) Define $\epsilon - \delta$ definition of continuity of a function at a point P also explain graphical meaning of continuity of a function.
- 6) Let f be real valued defined on subset $I \subseteq \mathbb{R}$. Let $P \in I$, if $\lim_{x \rightarrow P} f(x)$ exists then prove that it is unique

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