VCD - 9/10/17 MATHS I - FYBSC - SEM I ATKT EXAM - 75 MARKS - $2\frac{1}{2}$HRS

NOTE :

1) All questions are compulsory.

2) For Q.1, Q.2 and Q. 3 attempt any one subquestion (each 8 marks) from part (a), and any three subquestions (each 4 marks) from part (b).

3) For Q.4 , attempt any three.(each 5 marks)

Q.1. (a) Attempt any one. [each 8]

1) For $a, b, c \in IR$ prove the following

i) $a < b \Rightarrow -a > -b$

ii) $a < b \Rightarrow a + c < b + c$

iii) $a > b \,\&\, c < 0 \Rightarrow a < c$

iv) $a > b \,\&\, c > 0 \Rightarrow ac < bc$

2) State all properties of addition of IR = set of real numbers.

(b) Attempt any three. [each 4]

1) Find LUB and GLB of $S = \{1/n \,/\, n \in IN\}$

LUB= Least Upper Bound

GLB = Greatest Lower Bound

2) State and prove AM – GM inequality of IR.

3) State and prove Hausdorff property of neighbourhood.

4) Prove that if a_n is convergent sequence then it is bounded.

Q.2. (a) Attempt any one. [each 8]

1) State and prove Sandwich theorem for sequence. Hence discuss the convergence of $a_n = \frac{1}{n!}$

2) Define the Cauchy sequence in IR and show that following sequences are not Cauchy in IR

$i)\ a_n = (n^2) \quad n \in \mathbb{N} \qquad ii)\ a_n = 1 + \frac{1}{2} + \frac{1}{3} + \cdots \ldots\ldots\ldots + \frac{1}{n} \quad \forall n \in \mathbb{N}$

(b) Attempt any three. [each 4]

1) Examine whether the following sequences are monotonic

i) $a_n = (-1)^n$ ii) $a_n = \sin n$

2) Prove that if a_n & b_n are two convergent sequences having limits a and b respectively then prove that $(a_n + b_n)$ converges and $\lim_{n\to\infty}(a_n + b_n) == a + b$.

3) State and prove Cauchy completeness of IR.

4) Define a subsequence and prove that every bounded sequence in IR has convergent subsequence.

Q.3. (a) Attempt any one. [each 8]

1) Let f, g be real valued function defined on subset $J \subseteq IR$ and

$\lim_{x\to a} f(x) = l$, $\lim_{x\to a} g(x) = m$ then prove that

$\lim_{x\to a} f(x).g(x) = l.m.$

2) prove that $\lim_{x\to a} f(x) = l$ iff $\lim_{n\to\infty} f(x_n) = l$ for every sequence x_n converging to a $(x_n \neq a)$

(b) Attempt any three. [each 4]

1) Define the following functions

i) Constant function

ii) Absolute value function

2) Prove that Let f be real valued function on subset I of IR. Let $P \in I$, if $\lim_{x\to P} f(x)$ exists then it is unique.

3) Define right hand and left hand limits and evaluate right and left hand limits f as $\lim_{x\to P} f(x)$ in following case

$$f(x) = x - 1 \quad for\ x \leq 0$$
$$= 1 - x \quad for\ x > 0 \qquad \text{at } P = 0$$

4) Let $f, g : J \to IR$ be continuous at $P \in J$ where J is an open interval in IR then prove that f+g: $J \to IR$ continuous at P.

Q.4. Attempt any three. [each 5]

1) Define neighbourhood in IR and prove that intersection of any two neighbourhood $P \in IR$ is the neighbourhood of P.

2) Discuss the boundedness of $S = \{x/3 < x \leq 7\}$, $S = \{\sin x / x \in IR\}$

3) Show that every monotonic decreasing sequence is bounded above.

4) State all algebraic operations on sequences.

5) Define $\epsilon - \delta$ definiton of continuity of a function at a point P also explain graphical meaning of continuity of a function.

6) Let f be real valued defined on subset $I \subseteq IR$. Let $P \in I$, if $\lim_{x \to P} f(x)$ exists then prove that it is unique