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FYBSC- SEM I -MATHSII - $2\frac{1}{2}$HRS - 75 MARKS - V(D: 14/10/16 (ATKT-New

NOTE :

1) All questions are compulsory.

2) For Q.1, Q.2 and Q. 3 attempt any one subquestion (each 8 marks) from part (a). and any three subquestions (each 4 marks) from part (b).

3) For Q.4 , attempt any three.(each 5 marks)

Q.1. (a) Attempt any one. [each 8]

1) Define Euler φ function $\varphi(n)$ for positive integer $n \geq 1$ and prove that If $m, n \in \mathbb{Z}$ such that $(m,n) = 1$ then

$\varphi(m,n) = \varphi(m)\,\varphi(n)$ and also find $\varphi(390)$.

2) Find greatest common devisor of 2210, 357 and express it in form of $2810m + 457n$ $m, n \in \varphi$. Are m, n unique? Justify

(b) Attempt any three. [each 4]

1) State First principle of finite induction and prove that $8^n - 3^n$ is divisible by 5 $\forall\, n \in IN$.

2) Prove that

i) $2^n = n_{c_0} + n_{c_1} + n_{c_2} + \dots\dots\dots + n_{c_n}$

ii) $0 = n_{c_0} - n_{c_1} + n_{c_2} + \dots\dots\dots + (-1)^n n_{c_n}$

iii) $n_{c_0} + n_{c_2} + n_{c_4} + \dots\dots\dots = n_{c_1} + n_{c_3} + \dots\dots\dots = 2^{n-1}$

3) Using Pascal's triangle, expand $(a+b)^8$.

4) Define greatest common devisor of non zero integer a & b and prove that the positive gcd of any two integers (whenever exists) is unique.

Q.2. (a) Attempt any one. [each 8]

1) Define Invertible function, Bijective function and prove that $f : A \rightarrow B$ is invertible iff f is bijective.

2) Define i) Equivalence relation R on non empty set X

ii) partition of non empty set X.

and prove that if P is partition of non empty set X then P induces equivalence relation on X.

(b) Attempt any three. [each 4]

1) Check whether $*$ is binary on given set

i) $a * b = 5a + b$ on $\mathbb{Z}^+$

ii) $a + b = a \div b$ on IR $-\{0\}$

2) Determine whether each relation from A to B. If it is function give its range.

$A = \{a, b, c, d\}, B = \{6,7,8,9\}$

$g = [(a, 6), (b, 7), (c, 8), (d, 9)], h = \{(a, 6), (c, 7), (d, 7), (b, 9)\}$

3) Let $f : IR - \{3\} \to IR - \{0\}$ be defined by $f(x) = \frac{1}{x-3}$ then prove that f is bijective and Find formula for f^{-1}.

4) Determine whether following relation R on set A is equivalence or not

$a\,R\,b$ iff $a + b$ is even $\quad a, b \in \mathbb{Z} = A$.

Q.3. (a) Attempt any one. [each 8]

1) State and prove Remainder theorem for polynomial $f(x) \in F[x]$ and compute the remainder when $f(x)$ is divided by $g(x)$.

$f(x) = x^3 + 2x + 3$

$g(x) = x + 3$

2) State and prove Factor theorem for $f(x) \in F[x]$ Use it to determine whether or not $g(x)$ is factor of $f(x)$.

$f(x) = 3x^3 + 7x + 9\ ,\quad g(x) = x + 5$

(b) Attempt any three. [each 4]

1) Prove that a non constant polynomial $f(x) \in F[x]$ can be expressed as product of linear and quadradic polynomial.

2) Express $5i$ in polar form and also find magnitude and amplitude.

3) Find quotient and remainder when $f(x)$ is divided by $g(x)$

$$f(x) = x^3 - 6x^2 + 7x - 7$$

$$g(x) = x^2 + 3$$

4) Prove that $\sqrt{7}$, is irrational

Q.4. Attempt any three. [each 5]

1) Prove that two integers a and b are congruent modulo a positive integer iff a and b leave same remainder when divided by n.

2) Using Euler's theorem prove $5^{303} \equiv 4 (mod\ 11)$.

3) If R is an Equivalence relation on a non empty set X then prove that any two equivalence classes of X are either identical or disjoint.

4) Show that $f: A \to B$, $g: B \to C$ are function then for any nonempty subset X of A.

$$(gof)(X) = g(f(X))$$

5) State and prove Rational Root theorm.

6) Use De Moivre's theorem to prove that

$$\sin 3\theta = 3\cos^2\theta \sin\theta - \sin^3\theta$$

$$\cos 3\theta = \cos^3\theta - 3\cos\theta \sin^2\theta$$