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FYBSC- SEM I -MATHS I - $2\frac{1}{2}$ HRS - 75 MARKS - V CO: 13/10/16 ATKT C

NOTE :
1) All questions are compulsory.
2) For Q.1, Q.2 and Q. 3 attempt any one subquestion (each 8 marks) from part (a), and any three subquestions (each 4 marks) from part (b).
3) For Q.4 , attempt any three .(each 5)

Q.1. (a) Attempt any one. [each 8]

1) For $a, b, c \in IR$ prove the following

i) $a < b \Rightarrow -a > -b$  ii) $a < b \Rightarrow a + c < b + c$

iii) $a > b \& c > 0 \Rightarrow ac > bc$

2) state and prove Archimedian property of IR.

(b) Attempt any three. [each 4]

1) Show that If $\forall a, b \in IR$, $(ab)^{-1} = b^{-1}a^{-1} a \neq 0, b \neq 0$

2) Define absolute value and state properties of absolute value.

3) State and prove Hausdorff property of neighbourhood in IR

4) State and prove AM – GM inequality of IR.

Q.2. (a) Attempt any one. [each 8]

1) State all algebric properties of sequences.

2) Define the Cauchy sequence in IR and show that following sequences are not Cauchy in IR

$i)\ a_n = (n^2) \quad n \in \mathbb{N} \qquad ii)\ a_n = 1 + \frac{1}{2} + \frac{1}{3} + \cdots \ldots\ldots\ldots + \frac{1}{n} \forall n \in \mathbb{N}$

(b) Attempt any three. [each 4]

1) State and prove Sandwich theorem for sequence.

2) Show that every monotonic increasing sequence is bounded below.

3) Prove that if $a_n \& b_n$ are two convergent sequences having limits $a$ and b respectively then prove that $(a_n + b_n)$ converges and $\lim_{n\to\infty}(a_n + b_n) == a + b$.

Q.3. (a) Attempt any one. [each 8]

1) prove that $\lim_{x\to a} f(x) = l$ iff $\lim_{n\to\infty} f(x_n) = l$ for every sequence $x_n$ converging to $a$ $(x_n \neq a)$

2) Let $f, g$ be real valued function defined on subset $J \subseteq IR$ and

$\lim_{x\to a} f(x) = l$, $\lim_{x\to a} g(x) = m$ then prove that

$\lim_{x\to a} f(x) . g(x) = l\ m.$

(b) Attempt any three. [each 4]

1) Define the following functions and draw the graphs

i) Constant function ii) Identity function

2) Define right hand and left hand limits and evaluate right and left hand limits $f$ as $\lim_{x\to p} f(x)$ in following case

$f(x) = 2x$ if $x < 2$

$= x^2$ if $x > 2$ $p = 2$

3) Let $f, g : J \to IR$ be continuous at $P \in J$ where $J$ is an open interval in IR then prove that $f + g : J \to IR$ is continuous at P.

4) Prove that Let $f$ be real valued function on subset $I$ of IR. Let $P \in I$, if

$\lim_{x\to P} f(x)$ exists then it is unique.



Q.4. Attempt any three. [each 5]

1) Define $\epsilon - \delta$ definiton of continuity of a function at a point P also explain graphical meaning of continuity of a function.

2) Discuss the boundedness of $S = \{x/3 < x \leq 7\}$, $S = \{\sin x / x \in IR\}$

3) Prove that $\lim_{n\to\infty} \frac{a^n}{n!} = 0 \quad \forall a \in IR.$

4) Prove that $x \in IR^+$ then $\exists m \in IN$ such that $0 < \frac{1}{m} < x.$

5) Examine whether the following sequences are monotonic

i) $a_n = \frac{4}{n+1}$ ii) $a_n = \frac{4+3n}{n}$

6) Prove that if $a_n \& b_n$ are two convergent sequences having limits a and b respectively then prove that $(a_n + b_n)$ converges and $\lim_{n\to\infty}(a_n + b_n) = a + b.$