A-T.K.T F.Y.BSc – Maths – I Maths-I 1413116
Sem - I ~~Sem-II~~ 2015-16 ~~170~~

**Q.P. - Code :- SCIM010316X**

**Note :** 1) **All questions are compulsory.**

2) **For Q.1, Q.2, Q.3 Attempt any one subquestion (each 8 marks) from Part (a) and any three subquestion (each 4 marks) from Part (b).**

3) **For Q.4, attempt any three (each 5 marks)**

**Q. 1** A) **Attempt any one (Each 8 marks)**

1) For a, b, c∈IR, Prove following

i) $a<b \Rightarrow -a> -b$ ii) $a<b \Rightarrow a + c<b + c$

iii) $a>b, c<0 \Rightarrow ac<bc$ iv) $a>b$ & $c>0 \Rightarrow ac<bc$.

2) State and Prove Archimedian Property of IR.

B) **Attempt any three (Each 4 marks)**

1) Show that if ∀ a, b∈IR, $(ab)^{-1} = b^{-1} a^{-1}$ $a \neq 0$ , $b\neq 0$

2) Define bounded sequence of IR and prove that if $a_n$ is convergent then it is bounded.

3) State and Prove AM – GM inequality of IR.

4) Define Least Upper Bound (LUB) and Greatest Lower Bound (GLB) of a non empty Set A and find LUB and GLB of $A = \left\{\frac{1}{n} / n \in \underline{N}\right\}$

**Q. 2** A) **Attempt any one (Each 8 marks)**

1) State all algebraic properties of a sequences of IR.

2) Define the cauchy sequence in IR and show that following sequence are not cauchy in IR.

i) $a_n = (n^2)$ ∀ n∈IN ii) $1 + \frac{1}{2} + \frac{1}{3} + ... + \frac{1}{n}$ ∀ $n \in$ IN

B) **Attempt any three (Each 4 marks)**

1) Define Monotonic sequence in IR. Examine whether the following sequences are monotonic.

i) $a_n = (-1)^n$ ii) $a_n = \sin n$

2) Prove that if $(a_n)$ and $(b_n)$ are convergent sequences in IR such that $a_n \to a$, $b_n \to b$ respectively then prove that $(a_n + b_n) \to a + b$

3) Prove that if $a_n \to a$, α∈IR the $\alpha - a_n \to \alpha - a$

4) State and Prove Cauchy Completeness of IR.

**Q. 3 A) Attempt any one (Each 8 marks)**

1) Let f, g be real valued function defined on subset J of IR and $\lim_{x \to a} f(x) = l$, $\lim_{x \to a} g(x) = m$ then prove that $\lim_{x \to a} f(x) . g(x) = l . m$

2) Let $f : I \to R$ be a function where I is an open interval in IR Let $a \in I$ then prove that $\lim_{x \to a} f(x)$ exist iff $\lim_{x \to a+} f(x) = \lim_{x \to a-} f(x)$

**B) Attempt any three (Each 4 marks)**

1) Define right hand limits and left hand limits of f Evaluate right hand, left hand limits of f a $\lim_{x \to 3} f(x)$ in following case

$$f(x) = \begin{cases} x - 3 & \text{if } (x-3) \geq 0 \\ -(x-3) & \text{if } (x-3) < 0 \end{cases}$$

2) Give $\epsilon-\delta$ definition of limit of f at P and show that $\lim_{x \to 4} (2x + 3) = 11$

3) Prove that $f(x) = [x]$ is discontinuous at every integer.

4) Let f, g : $J \to IR$ be continuous at $P \in J$ where J is an open interval in IR Then Prove that $(f + g)$ is also continuous at P.

**Q. 4 Attempt any three (Each 5 marks)**

1) Let f be real valued function defined on subset J of IR Let $P \in J$, if $\lim_{x \to P} f(x)$ exists then prove that it is unique.

2) Define neighbourhood in IR and prove that intersection of any two neighbourhood $P \in IR$ is neighbourhood of P.

3) Discuss the boundedness of $S = \{x / 3 < x \leq 7\}$

$S = \{\sin x / x \in IR\}$

4) Define $\epsilon-\delta$ defination of continuity of a function at a point P and explain graphical meanin of continuity of function

5) Show that every monotonic decreasing sequence is bounded above.

6) Define the following function with example

i) Constant Function

ii) Absolute Value Function

**—— The End ——**