Note : 1) All questions are compulsory.

2) For Q.1, Q.2, Q.3 attempt any one sub question (Each 8 Marks) from Part (A) and Any three Subquestion (Each 4 marks) from Part (B).

3) For Q.4 attempt any three (Each 5 marks)

Q. 1 A) Attempt Any One : (Each 8 marks)

1) Define Euler ϕ function $\phi(n)$ for positive integer $n \geq 1$ and prove that if $m, n \in \mathbb{Z}$ such that gcd (m,n) = 1 then $\phi(m, n) = \phi(m)\phi(n)$ also find $\phi(580)$

2) Find greatest common divisor of 3120 and 750 and express it in form of $3120m + 750m$, $m,n \in \mathbb{Z}$. Are m,n unique ? Justify.

B) Attempt Any Three : (Each 4 marks)

1) Explain Pascal's Triangle. Use it to find $(a + b)^7$.

2) Prove that the number of primes are infinite.

3) Define least common multiple and greatest common divisor of non zero integer a and b. Prove that gcd (a, b) Lcm $[ab] = ab$.

4) State first principle of finite induction and prove that $8^n - 3^n$ is divisible by 5 $\forall n \in$ N.

Q. 2 A) Attempt Any One : (Each 8 marks)

1) Define invertible function, bejective function. Prove that composition of injective function is injective.

2) Define :

i) Equivalence relation R on nonempty set A

ii) Partition of A. Prove that every partition of a nonempty set A induces equivalence relation R on A.

B) Attempt Any Three : (Each 4 marks)

1) Check whether $*$ is binary on given set.

i) $a * b = a + b$ on IN. ii) $a * b = \min\{a, b\}$ on IR.

2) Lef F : IR $\rightarrow$ IR be defined by $f(x) = 3x - 7$. Check whether f is bijective or not. Hence find inverse if exist.

3) Determine whether following relation R on set A is equivalence a not.

A = list of relations among people.

R is defined as $(x, y) \in$ R if x is brother of y.

4) Determine whether each relation from A to B is function. If it is function give its range.

$A = \{a, b, c, d\}$ $B = \{2, 6, 8\}$

$F = \{(a, 2), (a, 6), (b, 6), (c, 6), (d, 8)\}$

$g = \{(a, 2), (b, 6), (c, 6), (d, 8)\}$

Q. 3 A) **Attempt Any One : (Each 8 marks)**

1) State and Prove Remainder Theorem for polynomial $f(x) \in f(x)$ and Computer remainder when $f(x)$ is divided by $g(x)$.

$f(x) = x^5 - 3x^4 + 4x^3 + x + 4$

$g(x) = x + 1$

2) State and Prove factor theorem. Use it to determine whether or not $g(x)$ is factor of $f(x)$.

$f(x) = x^4 + 4x^3 + 3x^2 + x + 5, g(x) = x + 2$

B) **Attempt Any Three : (Each 4 marks)**

1) Prove that a polynomial of degree n has at most n roots.

2) Express $\sqrt{3} + i$ in polar form, also find magnitude & amplitude.

3) Find quotient and remainder when f(x) is divided by g(x).

$f(x) = 5x^6 - 3x^2 + x + 1$

$g(x) = x^2 + x - 1$

4) Prove that a non constant polynomial $f(x) \in F(x)$ can be expressed as product of linear and quadratic polynomial.

Q. 4 **Attempt Any Three : (Each 5 marks)**

1) State and prove Euclid's Lemma.

2) Verify Wilson Theorem for P = 11.

3) Prove that $a \equiv b$ (modn) for a, b, $n \in Z^+$ iff a,b leave same remainder when divided by n.

4) If R is an equivalence relation on nonempty set X. Then Prove that any two equivalence classes of X are either identical or disjoint.

5) Use Demoivre's Theorem to prove that

$\sin 3\theta = 3\cos^2\theta \sin\theta - \sin^3\theta$.

$\cos 3\theta = \cos^3\theta - 3\cos\theta \sin^2\theta$.

6) State and prove Rational Root Theorem.

—— The End ——