Additional Exam

VCD – 230315

~~VCD~~ S.Y ITLDMS – SEM III – 75 MARKS – 2 ½ HRS

- Right indicates full marks
- All questions are compulsory

Q.1. Solve the following (Any Two) (10)

a) Find solution of R. R

$$a_n = 2a_{n-1} + 2a_{n-2} - 2a_{n-3} \quad a_0 = 9\,;\ a_1 = 10;\ a_2 = 32$$

b) Find solution of R.R and define Non- homogeneous R.R if

$$a_n = 4a_{n-1} - 4a_{n-2} + (n+1)2^n$$

c) Solve R. R by using generating function if

$$a_n = 4a_{n-1} - 3a_{n-2} \quad a_0 = 3 \ \&\ a_1 = 4.$$

d) Find 1st six terms of R.R if $a_n = 5a_{n-1} + 3 \quad a_1 = 7$ and define implicit formula.

Q.2. Solve the following (Any Two) (10)

a) Let R is an equivalence relation on set A let $a \in A \ \ \& b \in A \ \ aRb$ iff $R(a) = R(b)$.

b) If $R^* = \{(1,1)(1,2)(1,3)(1,4)(1,5)(2,2)(2,3)(2,4)(2,5)(3,3)(3,5)(4,4)(4,5)(5,5)\}$ then draw Hasse's digraph, check equivalence relation with reasons and find in-degree and out-degree of given set R.

c) Define poset and partially ordered relation and prove that $(Z^+/)$ is poset.

d) Show that $(D_{42}/)$ is lattice and find LUB and GLB.

Q.3. Solve the following (Any Two) (10)

a) Define composite function and let $f: A \to B, g: B \to C$ be invertible then gof is invertible.

b) A function $f: A \to B$ is invertible iff f is bijective.

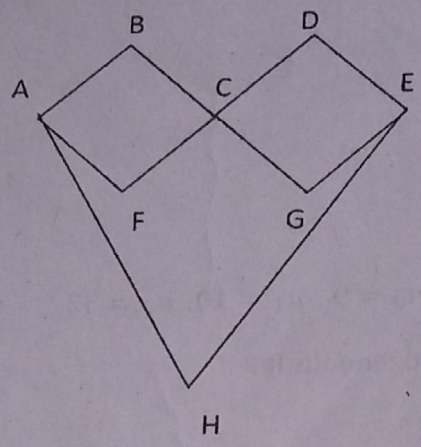
c) If $A = B = \{1,2,3,4\}$ where $f = \{(1,3)(2,1)(3,4)(4,3)\}$,
$g = \{(1,2)(2,3)(3,4)(4,1)\}$ find fog, gof, fof.

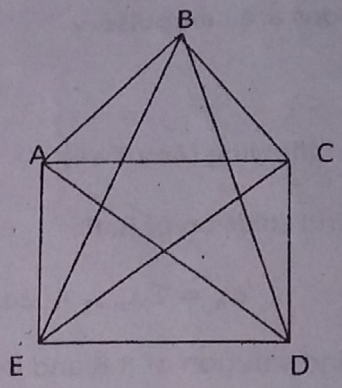
d) Define bijective function and inverse function and check givven function is bijective or not if $f(x) = \frac{4x-5}{3x-7} \quad A = R - [7/3], B = R - [4/3]$

Q.4. Solve the following (Any Two) (10)

a) Prove that the sum of degree of vertices in a graph is twice the number of edges in it.

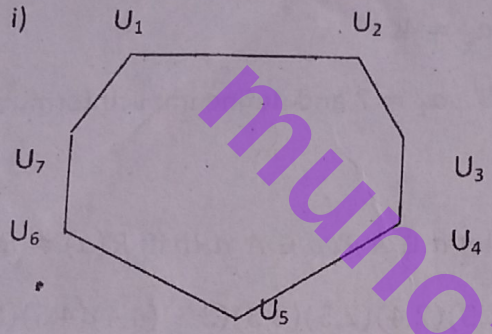
b) Define graph, path and circuit and check given digraphs are path,circuit & graph with explanation if



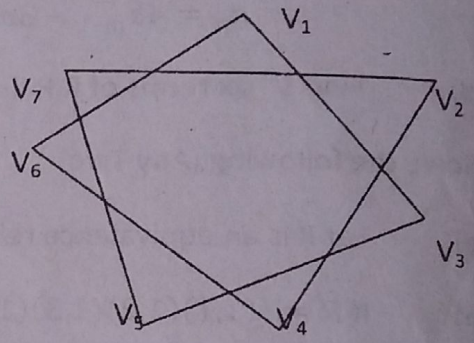


c) Show that graphs are Isomorphic graph and Define Isomorphic graph.

i)

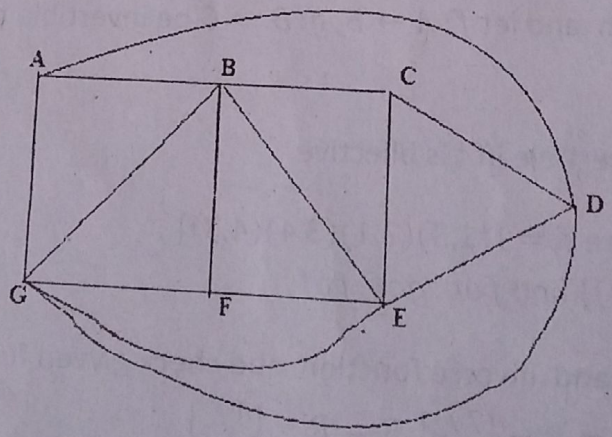


ii)



d) Construct tree diagram with the help of algebraic structure and find its value and draw spanning tree of following.

$$\left(3-\left(2\times 11-(9-3)\right)\right)\div\left(2+\left(3\times(7+4)\right)\right)$$



Q.5. Solve the following (Any Two) (10)

a) Let $G = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} / a, b, c, d \in G \quad ad - bc \neq 0 \right\}$ is a group w.r.t to usual multiplication of matrices.

b) Let (G*) and (G' *')be the two group and $F: G \to G'$ be homomorphism then prove that $f(e) = e'$ and $F(a^{-1}) = [F(a)]^{-1}$.

c) if $F: G \to G'$ be homomorphism with Kernel K then prove that K is normal subgroup of G.

d) prove that $a + b\sqrt{2} \in R \ \forall a, b \in R$ then R is ring with respect to addition.

Q.6. Solve the following (Any Two) (10)

a) using mathematical Induction show that $5 + 10 + 15 \ldots\ldots\ldots 5n = \frac{5n(n+1)}{2}$

b) Show that $p \wedge (q \vee r) = (p \wedge q) \vee (p \wedge r)$ with truth table and define Converse, Contrapositive and Inverse.

c) If A, B, C be any three sets then prove that $(A \setminus B) \times C = (A \times B) \cup (A \times C)$

d) Define Sets and its types with suitable example and explain set identities.

Q.7. Solve the following (Any Two) (15)

a) Find solution of R.R $a_{n+2} - a_{n+1} + 2a_n = n^2$, $a_0 = 1$ and $a_1 = -4$.

b) Let $A = \{a, b, c\}$ determine whether the relation R whose matrix M_R is given is an equivalence relation if $M_R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$ $M_R = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

c) List the ordered pairs in the relation R from $A = \{0,1,2,3,4,5\}$ to $B = \{0,1,2,3\}$ where $(a, b) \in R$.

d) Solve the following min spanning of tree by Prims algorithm and Krushical algorithm.

e) To prove that R/I with respect to addition is an abelian group.

f) Proved by mathematical induction that if A_1 A_2 ..... A_n are n- set then

$(\cap^{n}_{i=1} A_i) = \cup^{n}_{i=1} \overline{A_i}$