

(2½ hours)

Total Marks: 75

- N. B.: (1) **All** questions are **compulsory**.
 (2) Make **suitable assumptions** wherever necessary and **state the assumptions** made.
 (3) Answers to the **same question** must be **written together**.
 (4) Numbers to the **right** indicate **marks**.
 (5) Draw **neat labeled diagrams** wherever **necessary**.
 (6) Use of **Non-programmable** calculators is **allowed**.

1. Attempt any three of the following:

15

- a. Find the adjoint of the given matrix and hence find Inverse if exist

$$\begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$$

- b. Find the Characteristic values and characteristic vectors of the given matrix.

$$\begin{bmatrix} -17 & 18 & -6 \\ -18 & 19 & -6 \\ -9 & 9 & 2 \end{bmatrix}$$

- c. Discuss the consistency of the following systems of equations and solve them whenever possible.

$$X_1 + 2X_2 + 2X_3 = 1$$

$$2X_1 + 2X_2 + 3X_3 = 3$$

$$X_1 - X_2 + 3X_3 = 5$$

- d. Express in $a + ib$ form $\cot(x + iy)$.

- e. Solve the equation $x^7 + x^4 + x^3 + 1 = 0$.

- f. Prove that $(1 + \cos x + i \sin x)^n = 2^n \cos^n x/2 (\cos nx/2 + i \sin nx/2)$

2. Attempt any three of the following:

15

- a. Solve the Differential Equation $(1 - 2xy - x^3) dy - (1 + y^2 + 3x^2y) dx = 0$

- b. Solve the Differential Equation $x^2 dy/dx = 3x^2 - 2xy + 1$

- c. Solve the following Equation $\sec x \, dy/dx = y + \sin x$

- d. Solve the following Equation $p^2 x(x-2) + p(2y - 2xy - x + 2) + y^2 + y = 0$

- e. Find the Complementary and Particular Solution of the equation $(D^3 + D^2 + D + 1)y = \sin 2x$.

- f. Find the General Solution of the equation $(D^3 + 3D)y = \cos x$

3. Attempt any three of the following:

15

- a. Evaluate $\int_0^\infty e^{-3t} t \cos 2t \, dt$

- b. Find the inverse Laplace transform for the function

$$F(s) = \frac{5s+3}{(s-1)(s^2+2s+5)}$$

[TURN OVER]

- c. Find Laplace transformation of the function

$$f(t) = t(2\sin 3t + e^{2t})$$

- d. Obtain the Laplace transform of each of the given function

$$F(t) = e^{-2t}\cos 4t + e^{3t}\sin 6t$$

- e. Find Inverse Laplace Transformation by convolution theorem for

$$F(s) = \frac{s^2}{(s^2 + a^2)^2}$$

- f. Using Laplace transform method solve the following differential equations with the given condition.

$$(D^2 + 3D + 2)y = 4t + e^{3t} \text{ if } y = 1, Dy = -1 \text{ at } t = 0.$$

4. Attempt any three of the following:

15

a.

Evaluate $\int_0^2 \int_x^{4-x} \int_0^1 e^{2x+2y} dx dy.$

b.

Evaluate $\int_0^4 \int_0^{\sqrt{4x-x^2}} \frac{y dx dy}{(x^2 + y^2)^{1/2}}$

c.

Evaluate $\int_0^1 \int_{y^*y}^1 \int_0^{1-x} x dx dy dz.$

d.

Evaluate $\int_0^a \int_0^{(a^2-x^2)^{1/2}} \int_0^{(a^2-x^2-y^2)^{1/2}} (xyz) dx dy dz .$

e.

Change the order of integration and evaluate $\int_{-1}^2 \int_{x^2}^{x+2} dx dy$

f.

Change to polar coordinates and evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dx dy .$

5. Attempt any three of the following:

15

a.

Evaluate $\int_0^\pi \sin^2 x (1 + \cos x)^4 dx.$

b.

Evaluate $\int_0^\infty \frac{x^2 dx}{(1+x^6)^{7/2}}.$

[TURN OVER]

c.

Evaluate $\int_0^{\infty} e^{-ax} \sin x / x \, dx$

d.

Evaluate $\int_0^{\pi/2} \frac{\log(1+a \sin^2 x) dx}{\sin^2 x}$.

e.

Evaluate $\int_0^1 x^m (\log x)^n \, dx$.

f.

Define error function. Evaluate $\operatorname{erf}(\sqrt{x})$