

(Time: 2½ hours)

Total Marks: 75

- N. B.: (1) **All** questions are **compulsory**.
 (2) Make **suitable assumptions** wherever necessary and **state the assumptions** made.
 (3) Answers to the **same question** must be **written together**.
 (4) Numbers to the **right** indicate **marks**.
 (5) Draw **neat labeled diagrams** wherever **necessary**.
 (6) Use of **Non-programmable** calculators is **allowed**.

1. Attempt **any three** of the following:

15

- a. Show that $\begin{bmatrix} \frac{1}{\sqrt{3}} & 0 & \frac{2}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{6}} \\ \frac{-1}{\sqrt{3}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} \end{bmatrix}$ is an orthogonal matrix.
- b. For different values of k , discuss the following equations:
 $x + 2y - z = 0$; $3x + (k + 7)y - 3z = 0$; $2x + 4y + (k - 3)z = 0$
- c. Find the eigen values of the matrix
 $A = \begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$
- d. Express $\frac{-1}{2} + \frac{\sqrt{3}}{2}i$ in polar form.
- e. Prove that $(1 + i\sqrt{3})^8 + (1 - i\sqrt{3})^8 = -2^8$
- f. Show that $\sec^{-1}(\sin \theta) = \log \cot \left(\frac{\theta}{2} \right)$

2. Attempt **any three** of the following:

15

- a. Solve: $(D^2 - 4D + 1)y = \cos 2x + x$
- b. Solve $\sin 2x \frac{dy}{dx} = y + \tan x$
- c. Solve: $\frac{d^3 y}{dx^3} + \frac{d^2 y}{dx^2} - \frac{dy}{dx} - y = \cos 2x$
- d. Solve $p^2 - py + x = 0$
- e. Solve: $x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + 4y = \sin(\log x^2)$
- f. Solve: $\frac{du}{dx} + v = \sin x$; $\frac{dv}{dx} + u = \cos x$. given at $x = 0$, $u = 1$ and $v = 0$

[TURN OVER]

3. Attempt any three of the following:**15**a. Find the Laplace Transformation of $f(t) = t^3 e^{2t}$ b. $L[f(t)] = \frac{8 + 12s - 2s^2}{(s^2 + 4)^2}$ then find $L[f(2t)]$ c. Find $L[y(t)]$ of the following differential equation:

$$\frac{d^2 y}{dt^2} + 2 \frac{dy}{dt} + y = te^{-t}; \quad y(0) = 1 \text{ and } y'(0) = 2$$

d. Find the inverse Laplace transform of : $\frac{5s + 3}{(s + 1)(s^2 + 2s + 5)}$ e. Find the Laplace transform of : $f(t) = \begin{cases} 1 & 0 < t < a \\ -1 & a < t < 2a \end{cases}$ and $f(t) = f(t + 2a)$

f. Solve the following differential equation by using Laplace transform method:

$$\frac{d^2 y}{dt^2} + 2 \frac{dy}{dt} + 5y = e^{-t} \sin t. \text{ Given } y(0) = 0, y'(0) = 1$$

4. Attempt any three of the following:**15**a. Evaluate $\int_0^1 \int_0^1 \frac{dx dy}{\sqrt{(1-x^2)(1-y^2)}}$ b. Evaluate $\int_0^2 \int_0^{\sqrt{2x-x^2}} \frac{x dx dy}{\sqrt{x^2 + y^2}}$ by changing polar co-ordinates.c. Evaluate $\iint_R r^4 \cos^3 \theta dr d\theta$ where R is the region of curve $r = 2a \cos \theta$ d. Evaluate $\iiint \frac{dx dy dz}{\sqrt{1-x^2-y^2-z^2}}$ taken throughout the volume of the sphere $x^2 + y^2 + z^2 = 1$ in the positive octant.e. Evaluate $\iint y dx dy$ over the area bounded by $y = x^2, x + y = 2$ f. Find the volume bounded by the cylinder $y^2 = x$ and $x^2 = y^2$ and the planes $z = 0$ and $x + y + z = 1$ **5. Attempt any three of the following:****15**a. Evaluate $\int_0^{1/2} x^3 \sqrt{1-4x^2} dx$ b. Evaluate $\int_0^\pi \frac{\sin^4 \theta}{(1 + \cos \theta)^2} d\theta$ c. Show that: $\int_0^1 \frac{x^a - x^b}{\log x} \log \left(\frac{a+1}{b+1} \right) dx$ using DUIS.**[TURN OVER]**

- d. If $y = \int_0^x f(t) \sin [a(x-t)] \cdot dt$ then show that, $\frac{d^2 y}{dx^2} + a^2 y = af(x)$
- e. Find $\frac{d}{dx} [erf(x) + erf(ax)]$
- f. Define error function and prove that error function is an odd function.
