Note

- All questions are Compulsory
- Right indicates the full marks

Q.1 Attempt Any Three of the following   **(15)**

a) Find the Rank of the following Matrix by normal form $\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 2 & 3 & 1 & 1 \end{bmatrix}$

b) Test the consistency of the given system of equation

$$2x + 6y + 11 = 0, 6x + 20y - 6z + 3 = 0, 6y - 18z + 1 = 0.$$

c) Simplify $\dfrac{(\cos 5\theta + i \sin 5\theta)^3 (\cos 2\theta + i \sin 2\theta)^7}{(\cos 3\theta - i \sin 3\theta)^5}$

d) Verify the given Characteristic equation $A^3 - 5A^2 + 9A - I = 0$ if $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$

e) Examine whether the following vectors are linearly dependent or linearly independent

$$X_1 = [1,1,-1] \quad X_2 = [2,3,-5] \quad X_3 = [2,-1,4]$$

f) i) If $x + iy = \dfrac{1}{a+ib}$ then prove that $(x^2 + y^2)(a^2 + b^2) = 1.$

   ii) Write the following complex number in polar and exponential form if $z = \sqrt{3} + i$.

Q.2 Attempt Any Three of the following   **(15)**

a) Solve $\dfrac{dy}{dx} = \dfrac{4x - 3y + 1}{3x + 4y + 7}$

b) Solve $3e^x \tan y \, dx = (1 - e^x) \sec^2 y \, dy = 0$

c) Solve $(x + y)\dfrac{dy}{dx} + (x - y) = 0$ by substitution method.

d) Evaluate general solution of linear differential equation $\dfrac{dy}{dx} + y \cot x = \sin 2x.$

e) Evaluate complete solution of given differential equation if $\dfrac{d^2y}{dx^2} - 5\dfrac{dy}{dx} + 6y = e^{7x} + 8$

f) Solve the exact differential equation $(x^2 - x\tan^2 y + \sec^2 y)dy = (\tan y - 2xy - y)dx.$

Q.3 Attempt Any Three of the following   **(15)**

a) Find the value of given laplace $\int_0^\infty e^{-4t} t^2 \cos 5t \, dt$

b) Find $L^{-1}\left[\dfrac{1}{s^2 - s - 12}\right]$ by Convolution theorem

c) Find the Laplace transform of $L\left[t^2 e^{-4t} \cosh 2t\right]$

d) Using Laplace Transformation for the equation $\dfrac{dy}{dx} + 4y = 1 + e^{2t}$

e) Find the inverse Laplace transformation of $\frac{1}{(s^2+1)(s+4)}$

f) Find the laplace transform for following periodic function for period 2a

$$f(t) = \begin{cases} 4, & 0 \le t < 0 \\ -4, & a < t \le 2a \end{cases}$$

Q.4) Attempt Any Three of the following (15)

a) Solve $\int_0^5 \int_0^x x^2 y^2 \, dxdy$.

b) Solve $\int_0^\pi \int_0^{a(1+\cos\theta)} r \, dr d\theta$

c) Change the order of integration and evaluate $\int_0^2 \int_{2-\sqrt{4-y^2}}^{2+\sqrt{4+y^2}} dxdy$

d) Evaluate $\iint xy \, dxdy$ over the area bounded by parobolas $y = x^2, x = y^2$

e) Solve $\int_0^3 \int_0^x \int_0^y x^2 yz \, dxdydz$

f) Evaluate $\iiint x^2 yz \, dxdydz$ throughout the volume bounded by plane

$$x = 0, y = 0, z = 0, \quad \frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

Q.5 Attempt Any Three of following (15)

a) Solve $\int_0^1 (x\log x)^5 dx$

b) Solve $\int_0^\infty x^2 . e^{-x^4} dx$

c) Solve $\int_0^1 x^6 (1-x)^{1/2} dx$

d) Prove that $\text{erf}(x) + erf_c(x) = 1$

e) Prove that $\int_0^1 \frac{x^a - 1}{\log x} dx = \log(1+a)$

f) Evaluate $\int_0^\infty \frac{dx}{(1+x^2)^{9/2}}$