

$$\begin{cases} 0 & 0 \leq t < \frac{\pi}{2} \\ \cos t & \frac{\pi}{2} \leq t \leq \frac{3\pi}{2} \\ 0 & 0 \geq \frac{3\pi}{2} \end{cases}$$

e) Evaluate Laplace inverse by partial fraction if $L^{-1} \left[\frac{s+4}{s^2-1)(s+1)} \right]$

f) Find the laplace Transformation of given differential equation

$$3 \frac{dy}{dx} + 2y = e^{3t} \quad y = 1 \text{ at } t = 0.$$

Q.4) Attempt any three of the following

(15)

a) Evaluate $\int \int y \, dx \, dy$; $R : x = y$ and $y = x^2$

b) Evaluate $\int_0^1 \int_{x^2}^{2-x} y \, dx \, dy$.

c) Evaluate $\iiint \sqrt{1 - \frac{x^2}{a^2} - \frac{y^2}{b^2} - \frac{z^2}{c^2}} \, dx \, dy \, dz$ over the volume of an ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$.

d) Evaluate $\iiint (x + y + z) \, dx \, dy \, dz$ over the tetrahedron formed by $x=0$, $y=0$, $z=0$

e) Prove that $\int_0^1 \int_{y^2}^1 \int_0^{1-x} x \, dy \, dx \, dz = \frac{4}{35}$

f) Evaluate $\iiint \frac{dx \, dy \, dz}{(x^2 + y^2 + z^2)^{\frac{3}{2}}}$ over the annulus between the sphere

$$x^2 + y^2 + z^2 = a^2 \text{ and } x^2 + y^2 + z^2 = b^2 \quad a > b.$$

Q.5) Attempt any three of the following

(15)

a) Prove that $\operatorname{erf}(\infty) = 1$

b) Prove that $\int_0^\infty \frac{x^a - 1}{\log x} \, dx = \log(a + 1)$

c) Evaluate $\int_0^1 x^5 (1 - \sqrt{x})^7 \, dx$.

d) Prove that Reduction formula of Gamma function.

e) Show that $\int_0^{\frac{\pi}{2}} \sqrt{\tan \theta} \int_0^{\frac{\pi}{2}} \sqrt{\cot \theta} \, d\theta = \frac{\pi^2}{2}$.

f) Evaluate $\int_0^3 \frac{x^{\frac{3}{2}}}{(3-x)^2} \, dx$.

NOTE :

- All questions are Compulsory .
- Right Indicate the full marks .

Q.1) Attempt any three of the following

(15)

a) Find the Rank of the given matrix $A = \begin{bmatrix} 1 & 2 & 3 & 1 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \end{bmatrix}$ by Echelon form method .

b) Prove that following matrix is orthogonal and hence find A^{-1} if $A = \frac{1}{3} \begin{bmatrix} -2 & 1 & 2 \\ 2 & 2 & 1 \\ 1 & -2 & 2 \end{bmatrix}$

c) Verify Cayley's Hamilton Theroem if $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$

d) Simplify $\frac{(\cos 5\theta - i \sin 5\theta)^2 (\cos 7\theta + i \sin 7\theta)^{-3}}{(\cos 4\theta - i \sin 4\theta)^9 (\cos \theta + i \sin \theta)^5}$

e) If $5 \sinh x - \cosh x = 5$ find $\tanh x$

f) By considering principle value express $(1 + i\sqrt{3})^{(1+i\sqrt{3})}$ in the form of $a+ib$.

Q.2) Attempt any three of the following

(15)

a) Solve Non -exact differential equation if $(x - y^2)dx + 2xydy = 0$.

b) Solve $\frac{dy}{dx} = \frac{2x+2y+1}{3x+y-2}$

c) Solve Bernoulli's differential equation if $\frac{dy}{dx} = -xy + x^3y^3$

d) Find the general solution of the equation $(D^5 - D^4 - 2D^3 + 2D^2 + D - 1)y = e^x$

e) Solve $p^2 + px + py + xy = 0$

f) Solve $ayp^2 + (2x - b)p - y = 0$

Q.3) Attempt any three of the following

(15)

a) Prove that $L[\sin at] = \frac{a}{s^2 + a^2}$

b) Evaluate $L\left[\int_0^\infty e^{-4t} t^2 \cos t dt\right]$.

c) Find the inverse Laplace Transformation by using Convolution Theorem if $L^{-1}\left[\frac{s^2}{(s^2 - a^2)^2}\right]$

d) Express in Heaviside unit step function and find the Laplace transformation