Note -: *All questions are compulsary.

*Right indicates full marks.

Q.1) Solve the following (any two) **(10)**

a) Define Mathematical Induction. Prove by method of induction.

$p(n) = 1 + 5 + 9 + \cdots + (4n - 3) = (2n + 1)(2n - 1)$

Use (i)p(k) to show that p(k+1) (ii)Is p(n)true $\forall\, n \geq 1$

b) State and prove Inclusion and Exclusion principle and find how many number are not divisible by 2,3 and 5.

c) If p and q are true and r is false then prove the following truth values.How many number 1000's are not divisible by 2,5 and 9.

i) $p \leftrightarrow q = (p \wedge r) \vee (\sim p \wedge \sim q)$ ii) $(p \to r) \wedge (q \to r) = (p \vee q) \to r$

d) If A, B, c are subset of U then prove that $A \times (B \cup C) \subseteq (A \times B) \cup (A \times C)$ also give set identities.

Q.2) Solve the following (any two) **(10)**

a) Let R be the relation defined on Z such that xRy iff 5x+6y is divisible by 11 for $x, y \in z$.Show that R is equivalent relation.

b) If $(A \leq)$ and $(B \leq)$are poset then $(A \times B, \leq)$ is a poset with partial order $\leq$ defined by (a, b) $\leq$ (a' b') if $a \leq a' \in A$ and $b \leq b' \in B$.

c) A relation R is defined on

$A = \{1,2,3,4,5\}$ $R = \{(1,2)\ (1,6)\ (2,3)\ (3,3)\ (3,4)\ (4,2)\ (4,3)\ (4,5)\ (6,1)\ (6,4)\}$

Draw diagraph of R^{-1},R^2,R^c diagraph of M_R , M_R^2 ,Indegree,Outdegree and draw Hasses's diagraph.

d) Let L be the lattice then prove that

(i)$a \vee a = a$ $a \vee a = a$ (ii)$a \vee b = b \vee a$ $a \wedge b = b \wedge a$

(iii)$a \vee (b \vee c) = (a \vee b) \vee c$. $a \wedge (b \wedge c) = (a \wedge b) \wedge c$ (iv)$a \vee (a \wedge b) = a; a \wedge (a \wedge b) = a$

Q.3) Solve the following (any two) **(10)**

a) Define – i) Bijective function ii)Inverse function and find fog and gof if

$f(a) = a^2 - 1$ & $g(b) = \sqrt{b} + 1$.

b) Check given function is bijective or not if f(x) $= \frac{3x^2-4}{7x^2+8}$ also find inverse of it.

c) If function $f: A \to B$ is an invertible then prove that it is bijective.

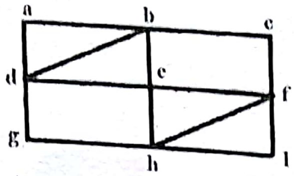
d) State and prove Pigeon hole Principal with suitable example. Also state Extended Pigeon hole principal with example.

P.T.O.

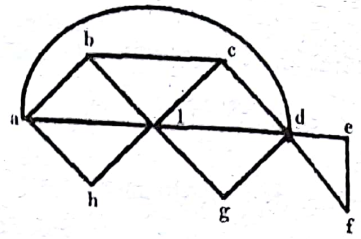
Q.4) Solve the following (any two)

a) Define – i) Graph ii) Minimal spanning tree and check the following digraph satisfy Eulerian and Hamiltonian path, graph and circuit. (10)

i)

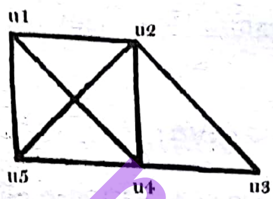


ii)

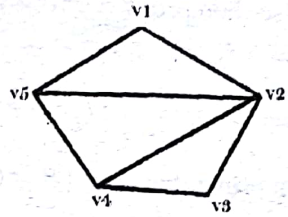


b) Define Isomorphic graph. Check following graphs are isomorphic.

i)



ii)

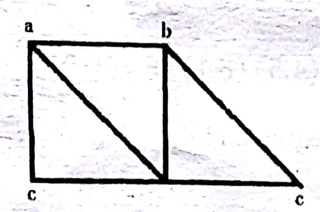


c) Construct tree and find value of tree. Also find pre-order, post order, Inorder and Left Data Right.

$$\left(M + \left(O - (T + K)\right)\right) \times \left(\left(G \div (S \times F)\right) \times W\right)$$

d) Prove that the number of veticese of an odd degree in a group is even and draw all possible spanning tree of given graph.

i)



ii)

Q.5) Solve the following (any two) (10)

a) Show that G = { 1,5,7,11,13,17 } with respect to multiplication modulo eighteen is group.

b) Prove that (3, 6) is a group code if $H = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ find minimum distance & how many e detects?

c) i) Prove that G= {1,-1,i,-i} is group under multiplication..

ii) (z_n *) is semi group for n=5 also show monoid for any n.

d)i) Let ℤ be a set of all even integers then show that semigroup (z +) and (T +) are isomorphic.

ii) Show that (z.×) isnot group.

Q.6) Solve the following (any two) **(10)**

a) Find the solution of Recurrnce Relation by using genratring function if $a_n - 7a_{n-1} + 8a_{n-2} = 0$, $a_0 = 2$, $a_1 = 5$.

b) Find the solution of Recurrnce Relation if $a_n = 2a_{n-1} + a_{n-2} - 2a_{n-3}$, $a_0 = 3, a_1 = 6, a_2 = 4$.

c) Find the solution of Recurrence Relation if $a_n = 8a_{n-1} - 16a_{n-2} + 2^n$, $a_0 = 1$, $a_1 = 3$.

d) Define Explicit formula and Implicit formula and Check $\{a_n\}$ is solution of Recurrence Relation if $a_n = 5a_{n-1} + 3^n$

$i)\ a_n = 0$ $ii)\ a_n = (-6)^n$ $iii)\ a_n = 3.(2)^n$ $iv)\ a_n = 4 + 2(2)^n$ $v)\ a_n = 1$.

Q.7) Solve the following (any three) **(15)**

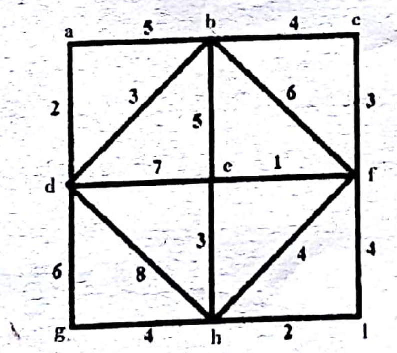
a) Let R be relation on set of real number s.t. xR_y iff x & y are real number differ by less than one i.e. $|x - y| < 1$. Show that R is equivalence relation.

b) Prove that $\frac{R}{I}$ w.r.t addition is an abalian group.

c) Let $f: A \to B, g: B \to C, h: C \to D$ find $(fog)oh)(-2), (fof)of(2), ((hoh)og)(-3)$ if $f(a) = x^2 + 2;\ g(b) = 5x, h(c) = 9x + 5$ and define composite function.

d) Define L.H.R.R and find 7[th] term of R.R if $a_n = 9a_{n-1} + 6a_{n-2} + 3^n$, $a_0 = 7$ $a_1 = 5$.

e) Apply the prims and Kruskal Algorithm. Find minimum weight of given diagram.



f) If A and B are two sets then verify following laws

i) Associative law ii) Distributive law with the help of Truth Table.

—— The End ——