Sem VCD- 26 09 15 LDMS- S.Y.BSc (I.T) -Sem -1-2015-16 - Marks-75 - Hrs-2 ½ 2015 - 16

Note -: All questions are compulsory.
Right indicates full marks.

Q.1) Solve the following (any two) **(10)**

a) Define Mathematical Induction. Prove by method of induction.

$$1^3 + 2^3 + 3^3 + \ldots\ldots\ldots + n^3 = \frac{n^3(n+1)^2+4}{4}.$$

b) State and prove Additional principal & How many numbers in N_{100} are multiple of 6 or 7?

c) With the help of truth table prove that equivalency.

i) $p \leftrightarrow q = (p \wedge r) \vee (\sim p \wedge \sim q)$ ii) $(p \to r) \wedge (q \to r) = (p \vee q) \to r$

d) If A, B, c are subset of U then prove that $A \times (B \cup C) \subseteq (A \times B) \cup (A \times C)$ also give set identities.

Q.2) Solve the following (any two) **(10)**

a) Let R be the equivalence relation of set A if $a \in A$ & $b \in A$ then a R b iff R(a) = R(b).

b) If (A <) and (B <) are poset then $(A \times B, \leq)$ is a poset with partial order ≤ defined by (a, b) ≤ (a' b') if $a \leq a' \in A$ and $b \leq b' \in B$.

c) Find Domain, Range, Indegree, Outdegree, digraph, M_R, M_{R^2}, M_S, M_{S^2}, $M_{R \cup S}$, $M_{R \cap S}$, R^{-1}.

$R = \{(a\ a), (a\ b), (a\ c), (b\ d), (b\ a), (b\ c), (b\ e), (c\ a), (c\ c), (c\ e), (d\ a), (d\ b), (d\ c), (d\ e), (e\ b), (e\ c), (e\ d)\}$

$S = \{(a\ a), (a\ c), (a\ e), (b\ b), (b\ d), (b\ e), (c\ a), (c\ c), (c\ c), (d\ a), (d\ b), (d\ c), (e\ a), (e\ b), (e\ c)\}$

d) Prove that $(L\ \leq)$ is poset and check $A = \{2,3,6,12,24,36,72\}$ is lattice.

Q.3) Solve the following (any two) **(10)**

a) Define – i) Bijective function ii) Inverse function and find fog and gof if

$f(x) = 2x^3 - 1$ & $g(x) = \left(\frac{x+1}{2}\right)^{1/3}$.

b) Define characteristic function and prove it's any two properties.

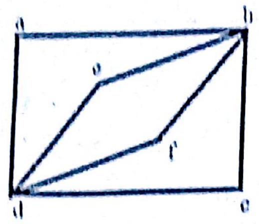
c) If function $f: A \to B$ is an invertible then prove that it is bijective.

d) State and prove Pigeon hole Principal with suitable example. Also state Extended Pigeon hole principal with example.

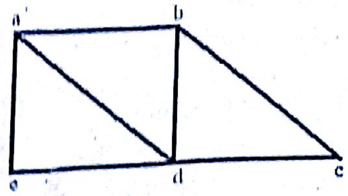
Q.4) Solve the following (any two) **(10)**

a) Define – i) Graph ii) Minimal spanning tree and check the following digraph satisfy Eulerian and Hamiltonian path, graph and circuit.

i)

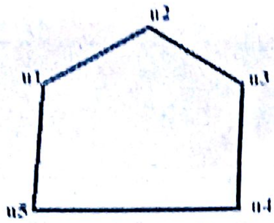


ii)



b) Define Isomorphic graph. Check following graphs are isomorphic.

i)



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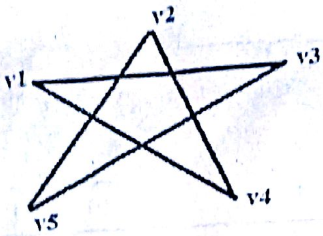
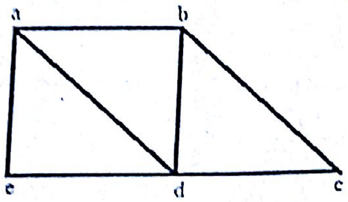


c) Construct tree and find value of tree. Also find pre-order, post order, Inorder and Left Data Right.

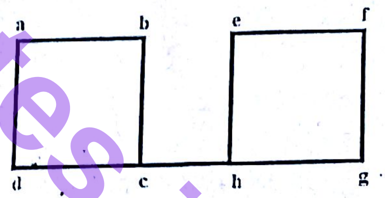
$$\left(x + \left(y - (x + y)\right)\right) \times \left(\left(3 \div (2 \times 7)\right) \times 4\right)$$

d) Prove that the number of vericers of odd degree in a group is even and draw all possible spanning tree of given graph.

i)



ii)



Q. 5) Solve any two. **(10)**

a) Show that prime residue classes modulo eleven with respect to multiplication modulo even.

b) Prove that (3, 6) is a group code if $H = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ find minimum distance & how many e detects?

c) i) Prove that every cyclic group is an abalian group.

ii) Let G be a group $ab \in G$ then prove that $(a^{-1})^{-1} = a$ & $(ab)^{-1} = b^{-1}.a^{-1}$.

d) Let $\mathbb{Z}$ be a set of all even integers then show that semigroup (z +) and (T +) are isomorphic.

Q. 6) Solve any two. **(10)**

a) Find the solution of Recurrncy Relation by using genratring function if $a_n - 5a_{n-1} + 6a_{n-2} = 0, a_0 = 6, a_1 = 30$.

b) Find the solution of Recurrncy Relation if $a_n = -3a_{n-1} - 3a_{n-2} - a_{n-3}, a_0 = 5, a_1 = -9, a_2 = 15$.

c) Find the solution of Recurrency Relation if $a_n = 7a_{n-1} - 6a_{n-2} + 3^n, a_0 = 1, a_1 = 3$.

d) Define Explicit formula and Implicit formula and Check $\{a_n\}$ is solution of Recurrency Relation if $a_n = 7a_{n-1} + 2^n$

$i)\ a_n = 0 \quad ii)\ a_n = (-6)^n \quad iii)\ a_n = 3.(2)^n \quad iv)\ a_n = 4 + 2(2)^n \quad v)\ a_n = 1$.

Q.7) Solve the following (any 3). **(15)**

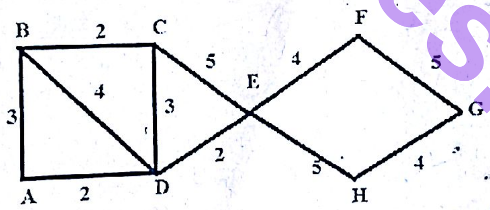
a) Show that R is an equivalence relation on set A iff $(a, b) \in R, (a, c) \in R$ then $(b, c) \in R$.

b) Prove that $\frac{R}{I}$ w.r.t addition is an abalian group.

c) Let $f: A \to B, g: B \to C, h: C \to D$ find $(fog)oh)(-2), (fof)of(2), ((hoh)og)(-3)$ if $f(a) = 2x^2 + 2,\ g(b) = 3x, h(c) = 9x + 5$ and define composite function.

d) Define L.H.R.R and find 7th term of R.R if $a_n = 9a_{n-1} + 6a_{n-2} + 3^n, a_0 = 2, a_1 = 1$.

e) Apply the prims and Kruskal Algorithm. Find minimum weight of given diagram.



f) If A and B are two sets then prove that

i) $\overline{A \cap B} = \bar{A} \cup \bar{B}$

ii) $\frac{A}{B} = A \cap B^C$