(Additional Exam) Additional

VCD – F.Y.I.T – Applied Maths II – SEM II 2015 – 75 MARKS – 2 ½ HRS

06/05/15

Q.1. Solve any two. [10]

a. If $a = cos2\alpha + isin2\alpha$, $b = cos2\beta + isin2\beta$, $c = cos2\gamma + isin2\gamma$

then show that $\sqrt{\frac{bc}{a}} + \sqrt{\frac{a}{bc}} = 2cos(\beta + \gamma - \alpha)$.

b. Prove that $\log\left(\frac{1}{1-e^{i\theta}}\right) = \log\left(\frac{1}{2}cosec\frac{\theta}{2}\right) + i\left(\frac{\pi}{2} - \frac{\theta}{2}\right)$.

C Find i) $tanhx$ if $sinhx - coshx = 5$. Ii) State&prove D-moiver's for the positive integer.

d. If α and β are root equation of $x^2 - \sqrt{3}x + 1 = 0$. Prove that $(\alpha)^n + (\beta)^n = 2cosn\frac{\pi}{6}$ and find $(\alpha)^{10} + (\beta)^{10}$.

Q.2. solve any two. [10]

a. Prove that $\lceil\overline{1} = 1$ and Prove that $\int_0^\infty \sqrt{y}\, e^{-y^2} dy = \frac{1}{2}\lceil\overline{\frac{3}{4}}$.

b. State and prove Duplication formula for Gamma function.

c. Show that $\int_{\pi/6a}^{\pi/2a} \frac{sinax}{x} dx = 0$ by Libnitz Theroem

d. Prove that i) $erf(\infty) = 1$ ii) $erf(x) + erfc(x) = 1$.

Q.3. solve any two. [10]

1. If $f(t) = coshat$ then prove that $L[coshat] = \frac{s}{s^2-a^2}$.

b. Solve by using convolution theorem $\frac{1}{(s-2)^4(s+3)}$.

c. Define Heavisides unit step function and find $f(t) = \begin{cases} 0 & 0 < t < \pi/2 \\ cos3t & \pi/2 < t < 3\pi/2 \\ 0 & t > 3\pi/2 \end{cases}$

d. Find $L[f(t)]$ if $f(t) = t \quad 0 < t < 1$

$f(t) = 0 \quad 1 < t < 2$

$f(t+2) = f(t) \quad t > 0$

$f(t)$ is periodic with period two

Q.4.solve any two. [10]

a.Find fourier series of

$$f(x) = \begin{cases} \pi x & 0 \le x \le 1 \\ \pi(2-x) & 1 \le x \le 2 \end{cases}$$

b.Find fourier series of $f(x) = |sinx| \in [-\pi, \pi]$.

c. Find fourier series of $f(x) = sinx \quad 0 \le x \le \pi$

$= 0 \quad \pi \le x \le 2\pi$.

d.Obtain Half Range Sine for $f(x) = e^x \in [0,1]$.

Q.5.solve any two. [10]

a.Evaluate $\int_0^{log2} \int_0^x \int_0^{x+y} e^{x+y+z} dx\, dydz$.

b.*Evaluate* $\iiint (x+y+z)\, dxdydz$ over the positve octant of sphere $x^2+y^2+z^2=a^2$.

c.*Show that* $\int_0^1 \int_0^{1-x} xy\sqrt{1-x-y}dy\, dx = \frac{16}{945}$.

d.Find $\iint xy^2 dxdy$ when R is the region bounded by $y = x^2, y = 0, x = 1$.

Q.6.solve any two. [10]

a.*Evaluate* $\int \frac{zsecz}{(1-z)^2} dz$ where C is the circle $|z| = 2$.

b. $F(z)=\frac{z}{z-1}$ at those singular point which lie inside IzI=2

c. Find the bilinear transformation that maps(1,i,-1) to (2,i,-2) z plane to w plane.

d. Find the image of Iz-2iI=2 under the mapping of $W=\frac{1}{z}$.

Q.7.solve any three. [15]

a. Prove that $\frac{sin6\theta}{\sin 2\theta} = 16cos^4\theta - 16cos^2\theta + 3$.

b. Evaluate $\int_0^\infty e^{-\sqrt{x}}.\sqrt{x}dx$.

c. Find $L^{-1}\left[\frac{2s+3}{s(s-1)(s+3)^2}\right]$.

d. Evaluate $\int_{-1}^{1} \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} log(x^2+y^2+1)\, dxdy$.

e. Find the Fourier series for $f(x) = x - x^2 \quad \in [-\pi, \pi]$.

f. State and prove Residue theorem. Also define analyitc function.
