VCD180423 SEM II FYIT NUMERICAL METHOD 75 MARKS 2 ½ HRS

Note

*** All questions are compulsory**

*** All questions carry equal marks**

*** Scientific Calculator is allowed**

Q.1) Attempt ANY THREE of the following. **(15)**

a) Suppose 1.414 is used as an approximation to √2. Find the absolute and relative errors.

b) Write short note on Conservation law of engineering problem.

c) Find the Truncation error in the expansion of $f(x) = e^{2x}$ evaluate first six terms in the series for x = 3.5

d) Explain blunders, formulation errors and data uncertainty.

e) Let $f(x) = x^3 - x^2 + x + 5$ at x = 2.45 using 3-digit arithmetic and determine the absolute & relative error using i) Rounding ii) Chopping.

f) Define -1) Significant digit 2) Error 3) Total numerical error 4) Round -off error 5) Error propagation.

Q.2) Attempt ANY THREE of the following. **(15)**

a) Using Secant Method, find the root of $f(x) = cosx - xe^x = 0$ taking the initial approximations as 0 and 1.

b) Find the smallest positive root of $f(x) = x^3 - 5x + 1 = 0$ by performing five iterations of Bisection Method.

c) Perform five iterations of Newton Raphson method to obtain the approximate value of equation, $x = 17^{\frac{1}{3}}$ starting with the initial approximation $x_0 = 2$.

d) For $f(x) = x - e^{-x} = 0$ determine the initial approximation to find the smallest positive root. Find the root correct to four decimal places using Regula False method up to four iterations.

e) Construct the divided difference table using Newton's Interpolation for the given data and hence find the interpolating polynomial.

x	0.5	1	1.5	2	2.5
F(x)	2	5	6	1	3

f) Solve by Lagrange's interpolation with the help of given data if f (1) = 3, f (3) = 5, f (5) = 9, f (7) = 2 then find f(4) .

Q.3) Attempt ANY THREE of the following. (15)

a) Solve the system 6x + y + z = 20, x + 4y − z = 6, x − y + 5z = 7 by using Gauss-Jordan Method.

b) Solve the system $5x + 3y + 9z = 2, 7x + 2y + Z = 3, x + 8y + z = 3$ by using Gauss Seidel Method.

c) From the data table given below obtain $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at x = 1 by Newton divided differentiation

x	1	1.2	1.4	1.6	1.8
y	2.7183	3.3201	4.0552	4.9530	6.0496

d) Solve by Trapezoidal rule if $\int_1^2 x^2\,dx$ dividing into six parts.

e) Solve by Simpson`s 1/3rd rule if $\int_0^1 \frac{1}{x^3+1}dx$ with h= 0.2

f) Evaluate $f'(2), f''(2)$ by Lagrange's interpolation differentiation with the help of given data

x	1	2	3	4
y	2	3	1	5

Q.4) Attempt ANY THREE of the following (15)

a) Solve by Simple Euler method if $\frac{dy}{dx} = x + 5y, y(0) = 1, find\ y\ at\ x = 0.2$ where h =0.2

b) Solve by Runge-Kutta forth order if $\frac{dy}{dx} = x^2 + 5y, y(0) = 1, find\ y\ at\ x = 0.5$ where h =0.5

c) c) Solve by Taylor's method up to fifth order derivative if $\frac{dy}{dx} = x^3 + 2x^2y + 1$

$y(1) = 1, find\ y\ at\ x = 2$ where h= 1.

d) Fit the equation of Straight line by Least Square method with the help of given data

x	1	2	3	4	5	6	7	8	9	10
y	0.2	0.4	0.5	0.6	0.8	1.2	1.4	1.6	1.8	2.1

e) Fit the equation of 2nd degree of polynomial by least square method with the help of given data

x	1	2	3	4	5	6	7	8	9
y	2	4	6	8	10	11	12	13	14

f) Evaluate equation X on Y and Y on X, $\bar{x}, \bar{y}, b_{xy}, b_{yx}$ if $9x + 3y = 16, 5x + 8y = 11$.

Q.5) Attempt ANY THREE of the following (15)

a) Maximize Z=6x+3y
subject to constraints, $2x + 3y \le 13$,
$x + y \le 5$
$x \ge 0, y \ge 0.$
Indicate the feasible region on graph and maximize the function Z = 6x+3y.

b) Give a mathematical formulation of the following L.P.P. The standard weight of a special purpose brick is 5 kg and it contains ingredients B_1 and B_2. B_1 costs Rs 5 per kg. and B_2 costs Rs 8 kg. Strength considerations dictate that the brick contains not more than 4 kg of B_1 and least 2 kg of B_2 Determine the amount of ingredients B_1 and B_2 so that the cost of the brick way be minimum Solve the problem graphically.

c) Find the solution of parabolic equation $\frac{\partial^2 u}{\partial x^2} = 2\frac{\partial u}{\partial t}$ given u(0,t) =0 , u(4,t) =0 u(x,0) =x (4-x) . Assume h =1 Find the values of u up to t = 5.

d) Using Crank -Nicholson Method, Solve the equation $u_{xx} = 14u_t$, subject to u(x,0) =0 , u(0,t) =0 and u(1,t) =$200t - 1$ .Compute t for one time step taking h=1/4 .

e) Classify the following equations in elliptic, parabolic and hyperbolic.

i) $(1 + x^2)u_{xx} - (5 + 2x^2)\frac{\partial^2 u}{\partial x \partial t} + (4 + x^2)\frac{\partial^2 u}{\partial t^2} = 0$

ii) $x^2\frac{\partial^2 u}{\partial x^2} + (1 - y^2)\frac{\partial^2 u}{\partial y^2} = 0 \quad , -\infty < x < \infty, -\infty < y < \infty$

iii) $\frac{\partial^2 u}{\partial x^2} + 4\frac{\partial^2 u}{\partial x \partial y} + 7\frac{\partial^2 u}{\partial y^2} - 2\frac{\partial u}{\partial x} + 6\frac{\partial u}{\partial y} - u = 0$

f) Solve the elliptic equation $u_{xx} + u_{yy} = 0$ for the following square mesh with boundary values as shown in figure by gauss Seidel iteration.