Q.P. Code: 33413

(2½ Hours)

[Total Marks: 75]

N. B.: (1) **All** questions are **compulsory**.
(2) Make **suitable assumptions** wherever necessary and **state the assumptions** made.
(3) Answers to the **same question** must be **written together**.
(4) Numbers to the **right** indicate **marks**.
(5) Draw **neat labelled diagrams** wherever **necessary**.
(6) Use of **Non-programmable** calculators is **allowed**.

1. Attempt *any three* of the following: **15**

a. State the characteristics of typical mathematical models of physical world. Explain with example.

b. Discuss the conservation laws and engineering with respect to mathematical models.

c. Suppose that you have the task of measuring the lengths of a bridge and a rivet and come up with 9999 and 9 cm, respectively. If the true values are 10,000 and 10 cm, respectively, compute (i) the true error and (ii) the true percent relative error for each case.

d. Use zero- through third-order Taylor series expansions to predict $f(3)$ for
$$f(x) = 25x^3 - 6x^2 + 7x - 88$$
using a base point at $x = 1$.

e. Determine the absolute and relative errors when approximating p by p^* when
i. $p = 0.3000 \times 10^1$ and $p^* = 0.3100 \times 10^1$
ii. $p = 0.3000 \times 10^{-3}$ and $p^* = 0.3100 \times 10^{-3}$
iii. $p = 0.3000 \times 10^4$ and $p^* = 0.3100 \times 10^4$

f. Let p = 0.54617 and q = 0.54601. Use four-digit arithmetic to approximate p − q and determine the absolute and relative errors using (i) rounding and (ii) chopping.

2. Attempt *any three* of the following: **15**

a. Use the Bisection method to find solutions accurate to within 10^{-2} for $x^3 - 7x^2 + 14x - 6 = 0$ in the interval [3.2,4].

b. The fourth-degree polynomial $f(x) = 230x^4 + 18x^3 + 9x^2 - 221x - 9$ in [0, 1]correct upto 4 decimal places using Regula-Falsi method.

c. Find the root of $4x^2 - e^x - e^{-x} = 0$ using Newton Raphson correct upto 4 decimal places using initial value as 1.

d. Given the cube of integers in the following table. Find the values of $(5.5)^3$ and 15^3 using Newton's interpolation formula.

e. Find $f(0.9)$ if $f(0.6) = -0.17694460$, $f(0.7) = 0.01375227$, $f(0.8) = 0.22363362$, $f(1.0) = 0.65809197$ using Lagrange's Interpolation formula.

f. Using appropriate interpolation formula find $f(4.25)$ from the table:

X	4.0	4.1	4.2	4.3	4.4	4.5
$f(x)$	27.21	30.18	33.35	36.06	40.73	54.01

[TURN OVER]

3. Attempt *any three* of the following: **15**

a. Solve the following system by using the Gauss-Jordan elimination method.

$$a + b + 2c = 1$$
$$2a - b + d = -2$$
$$a - b - c - 2d = 4$$
$$2a - b + 2c - d = 0$$

b. Solve the following system by using the Gauss-Seidel iterative method.

$$10a - b + 2c = 6$$
$$-a + 11b - c + 3d = 25$$
$$2a - b + 10c - d = -11$$
$$3b - c + 8d = 15$$

c. Find $\left(\frac{dy}{dx}\right)_{x=5.2}$, if

x	4.9	5.0	5.1	5.2	5.3	5.4	5.5
y	134.290	148.413	164.022	181.272	200.337	221.406	244.692

d. Evaluate $\int_0^{0.3} \sqrt{1 - 8x^2}\,dx$ using Simpson's 3/8th rule.

e. Apply Taylor's method of order two with $N = 10$ to the initial-value problem

$$y' = y - t^2 + 1, 0 \le t \le 2 \le, y(0) = 0.5$$

f. Using modified Euler's method find the solution of

$$y' = cos2t + sin3t), \quad 0 \le t \le 1; y(0) = 1 \ with\ h = 0.25$$

4. Attempt *any three* of the following: **15**

a. Fit an exponential model to:

x	0.4	0.8	1.2	1.6	2.0	2.3
y	800	975	1500	1950	2900	3600

b. Find the least square polynomial approximation of degree two to the data

x	0	1	2	3	4
y	–4	–1	4	11	20

c. Find the best-fit values of a and b so that $y = a + bx$ fits the data given in the table.

x	0	1	2	3	4
y	1	1.8	3.3	4.5	6.3

d. A painter has exactly 32 units of yellow dye and 54 units of green dye. He plans to mix as many gallons as possible of color A and color B. Each gallon of color A requires 4 units of yellow dye and 1 unit of green dye. Each gallon of color B requires 1 unit of yellow dye and 6 units of green dye. Find the maximum number of gallons he can mix graphically.

e. Rita wants to buy x oranges and y peaches from the store. She must buy at least 5 oranges and the number of oranges must be less than twice the number of peaches. An orange weighs150 grams and a peach weighs 100 grams. Joanne can carry not more than 3.6 kg of fruits home.

i) Write 3 inequalities to represent the information given above.

ii) Plot the inequalities on the Cartesian grid and show the region that satisfies all the inequalities. Label the region S.

iii) Oranges cost ₹ 0.70 each and peaches cost ₹ 0.90 each. Find the maximum that Rita can spend buying the fruits.

[TURN OVER]

f. Consider a calculator company which produces a scientific calculator and a graphing calculator. Long-term projections indicate an expected demand of at least 1000 scientific and 800 graphing calculators each month. Because of limitations on production capacity, no more than 2000 scientific and 1700 graphing calculators can be made monthly. To satisfy a supplying contract, a total of atleast 2000 calculators must be supplied each month. If each scientific calculator sold results in Rs.120 profit and each graphing calculator sold produces a Rs.150 profit, how many of each type of calculators should be made monthly to maximize the net profit?

5. Attempt *any three* of the following: **15**

a. The mileage C in thousands of miles which car owners get with a certain kind of tyre is a random variable having probability density function

$$f(x) = \frac{1}{20} e^{-\frac{x}{20}} \quad for\ x > 0$$
$$= 0, \quad for\ x \leq 0$$

Find the probabilities that one of these tyres will last

i. At most 10000 miles
ii. Anywhere from 16000 to 24000 miles
iii. At least 30000 miles

b. A petrol pump is supplied with petrol once a day. If its daily volume *X* of sales in thousands of litres is distributed by

$$f(x) = 5(1-x)^4, 0 \leq x \leq 1$$

what must be the capacity of its tank in order that the probability that its supply will be exhausted in a given day shall be 0.01?

c. A continuous random variable X has a p.d.f.

$$f(x) = 3x^2, \ 0 \leq x \leq 1$$

Find *a* and *b* such that

i. $P(X \leq a) = P(X > a)$ and
ii. $P(X > b) = 0.05$

d. What is the probability of getting a total of 9 (i) twice and (ii) at least twice in 6 tosses of a pair of dice?

e. In a precision bombing attack there is a 50% chance that any one bomb will strike the target. Two direct hits are required to destroy the target completely. How many bombs must be dropped to give a 99% chance or better of completely destroying the target?

f. A car hire firm has two cars which it fires out day by day. The number of demands for a car on each day is distributed as Poisson variate with mean 1.5. Calculate the proportion of days on which (i) neither car is used, and (ii) some demand is refused.