

Q.1.Solve any two.

[10]

- Convert into Polar form $Z = \frac{1}{2} - \frac{\sqrt{3}}{2}i$ and also prove first case of De Moivre's
- Prove that i) $\left(\frac{1+\tanh x}{1-\tanh x}\right)^n = \cosh 2nx + \sinh 2nx$.
- Find the value of $\log(\sin(x + iy))$.
- Express $\frac{1+2i}{(1+i)(1-3i)}$ in the form of $a+ib$.

Q.2.Solve any two.

[10]

- Prove that $|1| = 1$ Also find $\int_0^\infty e^{-4x^2} dx$
- Evaluate $\int_0^\infty e^{-4x} x^7 dx$
- Show that $\int_0^1 \frac{e^{ax}-e^{bx}}{x} dx = \log\left(\frac{b}{a}\right)$
- Prove that i) $\operatorname{erfc}(-x) + \operatorname{erfc}(x) = 2$
ii) Prove that error function is an odd function

Q.3.Solve any two.

[10]

- Prove that $L[1] = \frac{1}{s}$ 2) $L[\cosh 5t \sin 2t]$
- Solve by using convolution formula If $F(s) = \frac{1}{(s-2)(s-1)^2}$
- Find the Periodic Function with period T . If $f(t) = 1$ for $0 \leq t \leq a$ $f(t) = -1$ $a \leq t \leq 2a$.
- Find Heaviside Unit step function if $f(t) = t^3$ $0 < t < 1$
 $= 4t$ $t > 1$

Q.4 Solve any two

[10]

- Find Fourier series for $f(x) = \begin{cases} x & 0 \leq x \leq \pi \\ 2\pi + x & \pi \leq x \leq 2\pi \end{cases}$
- Show that expansion of $f(x)$ in F.S. $f(x) = x + 2 \in [-\pi, \pi]$
- Find Fourier series of $f(x) = 1 - x^2 \in [0, \pi]$.
- Obtain Fourier series for function for $f(x) = x + 1$ $-2 \leq x \leq 0$.
 $= x$ $0 \leq x \leq 2$

Q.5. Solve any two

a. Evaluate $\int_0^a \int_0^{1+\sin\theta} r^2 \cos\theta \, dr \, d\theta$

b. Evaluate $\iiint (x^3 y^2 + y^2 z^2 + z^3 x^2) \, dx \, dy \, dz$ throughout the volume of sphere $x^2 + y^2 + z^2 = 4$.

c. Show that $\int_0^1 \int_0^{\sqrt{1-x^2}} 4x \, dx \, dy = \frac{4}{3}$.

D Find the volume of paraboloid of revolution $x^2 + y^2 = 4z$ and cut of the plane $z=4$

Q.6. Solve any two.

a. Evaluate $\int \frac{z \cos \pi z}{z^2 - z - 2} \, dz$ where C is the $|z + i| = 2$.

b. Find residue of $\frac{z^3}{(z-1)(z+2)(z+4)}$ at Pole and show that their sum is zero.

c. Find the bilinear transformation that maps $(1, i, -1)$ to $(2, i, -2)$ z plane to w plane.

d. Find the image of $|z-2i|=2$ under the mapping of $W = \frac{1}{z}$.

Q.7. Solve any three.

a. Express $\cos^7 \theta$ in a multiple of θ

b. State and prove Beta Gamma relation.

c. Find Laplace if $\frac{dy}{dt} + 2y = \cos t$, $y(0) = 1$.

d. Evaluate $\int_0^1 \int_{x^2}^x (x^2 + 3y + 2) \, dx \, dy$.

e. Find the Fourier series for $f(x) = x$ for $[-\pi, \pi]$.

f. State and prove Residue theorem.