FYIT
Sem II

[redacted] F.Y.I.T – Applied Maths II – SEM II 2015 – 75 MARKS – 2 ½HRS

VCD130315

Date - 13/3/15

Q.1.Solve any two. [10]

a. If $sin\alpha = i\ tan\alpha$ then show that $cos\theta + isin\theta = tan\left(\frac{\pi}{4} + \frac{\alpha}{2}\right)$

b. If $p\log(a+ib) = (x+iy)\log m$. prove that $\frac{y}{x} = \frac{\tan^{-1}\frac{b}{a}}{\log(a^2+b^2)}$.

c. Prove that $\cos h^7 x = \frac{1}{64}[cosh7x + 7cosh5x + 21cosh3x + 35coshx]$

d. Find z if $arg(z+2i) = \frac{\pi}{4}$ and $arg(z-2i) = \frac{3\pi}{4}$.

Q.2.Solve any two. [10]

a.Prove that $\lfloor n = 2\int_0^\infty e^{-x^2}.x^{2n-1}$. Also find $\int_0^\infty \frac{x^4}{4^x}dx = \frac{24}{(log4)^3}$.

b.State and prove Duplication formula for Gamma function.

c.Show that $\int_0^\infty \frac{cos\lambda x}{x}(e^{-ax} - e^{-bx})dx = \frac{1}{2}\log\left(\frac{b^2+\lambda^2}{a^2+\lambda^2}\right)$.

d. Prove that i) Error function is an odd function.

ii) $erfc(x) + erfc(-x) = 2$.

Q.3.Solve any two. [10]

a. If $f(t) = e^{-at}$ then prove that $L[e^{at}] = \frac{1}{s-a}$ and also find $L[t^2cosh7t]$

b. Solve by using convolution formula If F(s)=$\frac{1}{(s-2)(s-2)^2}$.

c. Find the Periodic Fuction with period T. If $f(t) = l\frac{t}{T}$ for $0 < t < T$ $f(t) = f(t+T)$.

d. Find Heavy SideUnit step function if $f(t) = t \quad 0 < t < 2$

$= t^2 \quad t > 2$.

Q.4.Solve any two. [10]

a. Find Fourier series for $f(x) = \begin{cases} x & -1 \le x \le 0 \\ 2+x & 0 \le x \le 1 \end{cases}$

b. Find Fourier series for $f(x) = e^{-x} \in [0, 2\pi]$.

c. Find Fourier series of $f(x) = cosax \in [-\pi, \pi]$.

d. Obtain Half Range Sine function for $f(x) = \frac{1}{4} - x \quad 0 < x < \frac{1}{2}$

$= x - \frac{3}{4} \quad \frac{1}{2} < x < 1$.

Q.5.Solve any two. [10]

a. Evaluate $\int_1^2 \int_2^3 \int_1^3 (x^2y + z)dx\,dydz$.

b. Evaluate $\iiint (x^2y^2 + y^2z^2 + z^2x^2)\,dxdydz$ throughout the volume of sphere $x^2 + y^2 + z^2 = a^2$

c. $Show\ that \int_0^1 \int_0^y xye^{-x^2} dx\,dy = \frac{1}{4e}$.

d. Find $\iint y\,dxdy$ over area bounded by $y = x^2$ & $x + y = 2$.

Q.6.Solve any two. [10]

a. Evaiuate $\int \frac{ze^{2z}}{(z-1)^3} dz$where C is the$|z + i| = 2$.

b. Find residue of$\frac{z^3}{(z-1)(z-2)(z-3)}$ at 1, 2, 3 and infinity and show that their sum is zero.

c. Find the Bilinear transformation which maps the point$z = 1, i, -1$ into the points $w = i, 0, i$ and find image of $|z| < 1$.

d. Find the image of circle $(x-3)^2 + y^2=2$ under the transformation $W=\frac{1}{z}$

Q.7.Solve any three. [15]

a. Prove that $\frac{sin7\theta}{sin\theta} = 7 - 56sin^2\theta + 112sin^4\theta - 64sin^4\theta$.

b. Evaluate $\int_0^\infty \left(\frac{x}{1+x^2}\right)^6 dx$.

c. Find Laplace if $\frac{dy}{dt} + y = \cos 2t, y(0) = 1$.

d. Evaluate $\int_0^a \int_0^{\sqrt{a^2-y^2}} (x^2 + y^2)\,dxdy$.

e. Find the Fourier series for even function for $[-\pi, \pi]$.

f. State and prove Residue theorem. Also define analyitc function

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