FYIT Sem I
Date. 23/3/15

VCD – 230315 FYIT APPLIED MATHS I – SEM I – 75 MARKS – 2 ½ HRS

- Right indicates full marks
- All questions are compulsory

Q.1. Solve the following. (any two) (10)

a) Find A^{-1} by Inversion method

$$\text{If } A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 2 & -3 \\ 1 & -4 & 9 \end{bmatrix}$$

b) Check following vectors are linearly Dependent or Independent if

$[1 \quad 2 \quad -1 \quad 0]$ $[1 \quad 3 \quad 1 \quad 2]$ $[4 \quad 2 \quad 1 \quad 0]$ $[6 \quad 1 \quad 0 \quad 1]$

c) Test for consistency the following equation and if possible solve them.

$x_1 - x_2 + 2x_3 + x_4 = 2$; $3x_1 + 2x_2 + x_4 = 1$; $4x_1 + x_2 + 2x_3 + 2x_4 = 3$

d) Find Rank of following matrix (i) by normal form and (ii) by echelon form

i) $\begin{bmatrix} 2 & 1 & 4 & -1 \\ 1 & 2 & 1 & 3 \\ 4 & 5 & -1 & 2 \\ 8 & 7 & 7 & 3 \end{bmatrix}$ ii) $\begin{bmatrix} 2 & -1 & 3 & 4 \\ 0 & 3 & 4 & 1 \\ 2 & 3 & 7 & 4 \\ 2 & 5 & 11 & 6 \end{bmatrix}$

Q.2. Solve the following. (any two) (10)

a) Find Eigen value and Eigen vector of following matrix

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 2 & 3 & 2 \\ 3 & 3 & 4 \end{bmatrix}$$

b) Verify Caley's theorem

$$A = \begin{bmatrix} 7 & -1 & 3 \\ 6 & 1 & 4 \\ 2 & 4 & 8 \end{bmatrix}$$

c) If $A = \begin{bmatrix} 1/3 & 2/3 & a \\ 2/3 & 1/3 & b \\ 2/3 & -2/3 & c \end{bmatrix}$ is orthogonal and also find A^{-1}.

d) Express in $p + iQ$ where p is a real symmetric and Q is a real skew symmetric if

$$A = \begin{bmatrix} 3i & -1+i & 3-2i \\ 1+i & -i & 1+2i \\ -3-2i & -1+2i & 0 \end{bmatrix}$$

Q.3. Solve the following. (any two) (10)

a) $y^2 + x^2 \frac{dy}{dx} = xy \frac{dy}{dx}$

b) $(2x + 3y - 1)dx + (3x - 2y + 1)dy = 0$

c) $(1 + x^2)\frac{dy}{dx} + y = e^{\tan^{-1}x}$

d) State and prove Bernoulli's theorem.

Q.4. Solve the following. (any two) (10)

a) Find n^{th} derivative of $\frac{x}{(x-1)(x-2)(x-3)}$.

b) If $u = e^{xyz}$ then prove that $\frac{\partial^2 u}{\partial x\, \partial y\, \partial z} = (1 + 3xyz + x^2y^2z^2)e^{xyz}$.

c) State and prove Rolle's theorem and verify Rolle's theorem if $f(x) = x^2 - 4 \;\; in \; [0,4]$.

d) Find maximum and minimum value of $f(x,y)$ if $f(x,y) = y^2 + 4xy + 3x^2 + x^3$

Q.5. Solve the following. (any two) (10)

a) Calculate $\nabla^2 f$ when $f = 3x^2z - y^2z^3 + 4x^3y + 2x - 3y - 5$ at point (1,1,0).

b) Prove that $\bar{A} = (6xy + z^3)\bar{i} + (3x^2 - z)\bar{j} + (3xz^2 - y)\bar{k}$ is irrotational and find φ such that $\bar{A} = \nabla\varphi$.

c) Find angle between the normal to the surfaces $xy = z^2$ at $P(1,1,1)$ & $Q(4,1,2)$.

d) Find Curl $(curl\, \bar{A}) = x^3y\bar{i} - 2x^2z\bar{j} + 3yz\bar{k}$ at (1,3,4).

Q.6. Solve the following. (any two) (10)

a) Solve $(D^3 - 2D^2 - 5D + 6)y = 0 \;\; y(0) = 0, y'(0) = 0, y''(0) = 1$.

b) Solve $(D^6 - 64)y = e^x \cos h2x$.

c) Solve $(D^2 + 3D + 2)y = \sin 2x$.

d) Solve $(D^2 - 2D + 5)y = 25x^2 + 12$.

7. Solve the following. (any three) (15)

a) Verify Euler's theorem $u = \frac{x+y+z}{\sqrt{x}+\sqrt{y}+\sqrt{z}}$

b) Find A^{-1} by adjoint method if $A = \begin{bmatrix} 1 & 5 & 3 \\ 2 & 1 & 3 \\ 2 & 5 & 2 \end{bmatrix}$.

c) If $\bar{A} = 2xyz\bar{i} - x^2yj + xz^2k$, $\bar{B} = x^2\bar{i} + yz\bar{j} - xy\bar{k}$ & $\varphi = 2x^2yz^3$ then find
i) $(\bar{A}.\nabla)\varphi$ ii) $(\bar{B}.\nabla)A$ iii) $\bar{A}.\nabla\varphi$

d) Solve $xe^x(dx - dy) + e^x dx + ye^y dy = 0$ check exact if not then apply particular formula and find general solution.

e) Check the given matrix is Derogatory or non derogatory

$$\begin{bmatrix} 7 & 4 & -1 \\ 4 & 7 & -1 \\ -4 & -4 & 4 \end{bmatrix}$$

f) Solve i) $(D^4 - 6D^3 + 12D^2 - 8D)y = 0$

ii) $2D^3y - 2Dy - y = 0$