

25/11/19

VCD- F.Y.BSc (IT) DISCRETE MATHEMATICS SEM-I MARKS-75 TIME-2<sup>1/2</sup> Hrs.

Note- \*Right indicate full Marks

\*All questions are compulsory.

Q.1) Solve any three of the following

(15)

- Check the argument is valid or invalid If I got Salary I'll buy T.V. If I sell my Scooty I'll buy T.V. Therefore If I got salary or I sell my scooty then I 'll buy a T.V.
- Let  $U = \{1, 2, 3, \dots, 10\}$ ,  $X = \{1, 2, 3, 4, 5\}$ ,  $Y = \{y/y=2x \in X\}$ ,  $Z = \{z/z^2-9z+15=0\}$  L L  
Find i)  $X \cap Y$  ii)  $Y \cup Z$  iii)  $X-Z$  iv)  $Y^c$  v)  $X^c-Z^c$  vi)  $(X-Z)^c$
- Prove that  $(A \cap B) \times (C \cap D) = (A \times C) \cap (B \times D)$
- In a college 100 students have access to 3 software packages A,B,C.28 did not use software, 8 use only package A, 26 used only packages B, 7 used packages C, 10 used all the three packages, 13 used both A and B. Draw a Venn diagram with all sets enumerated for as possible. Label the two subsets which cannot be enumerated as x and y.  
i) If twice as many students use package B as package A. Write down the pair of simultaneous equation in x and y. find x and y.  
ii) How many students used package C?
- Write algebraic proof of the  $(A \cup B) - (C - A) = A \cup (B - C)$
- Simplify and show that i)  $[(x_1 \cdot x_2) + x_3] \cdot x_1 = x_3 \cdot x_1$  ii)  $(x \cdot y) + (x \cdot z) + (x \cdot z') + (x \cdot p) = x$

Q.2) Solve any three of the following

(15)

- Prove that  $1+3\sqrt{2}$  is irrational.
- Define Euclidian Algorithm and prove that for all real number x and for all integer m,  $[x + m] = [x] + m$
- Prove that if  $5n+2$  is odd then n is odd.
- Define Quotient Remainder theorem and prove that for all x and y belong to  $\mathbb{Z}$  if  $x \bmod 7 = 5$  and  $y \bmod 7 = 6$  then  $xy \bmod 7 = 2$
- Prove that the difference of any two rational number is rational number.
- Prove that for all integer m and n, m+n and m-n are either both even or odd.

Q.3) Solve any three of the following

(15)

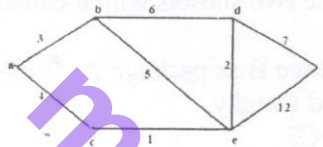
- Show that  $n \geq 3 \Rightarrow f_n > \alpha^{n-2}$  where  $\alpha = 1 + \sqrt{5}/2$  by strong induction
- Let  $A_1, A_2, \dots, A_n$  be any n sets we show by Mathematical induction that  $(\bigcup_{i=1}^n A_i) = \bigcap A_i$

- c) Prove that if the given predicate is true before entering loop it is true after entering loop  
loop: while ( $m \geq 0$  and  $m \leq 100$ )  
 $m := 3.m$   
 $n := 5.n$   
end while  $m^3 > n^2$
- d) Define Recursive relation and find explicit formula if  $r_k = 9r_{k-1}$  for all integer  $n \geq 2$ ,  $r_0 = 4$ ,  $r_1 = 3$
- e) Define and give suitable example of each – i) Range ii) Constant function  
iii) Logarithmic function iv) Encoding and Decoding function.
- f) If  $A = \{0, 1, 2, 3, 4\}$  and define  $f: A \rightarrow A$  and  $g: A \rightarrow A$  such that  
 $x \in A$   $f(x) = (x + 4)^2 \bmod 4$  and  
 $g(x) = x^2 + 3x + 1 \bmod 5$  Does  $F = g$ ? explain.

**Q.4) Solve any three of the following**

(15)

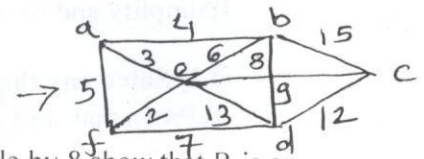
- a) Find shortest path by using Dijkstra's Algorithm



- b) Define and give counter example of each Simple graph, Pseudo graph, circuit, Trail, Degree of vertex.

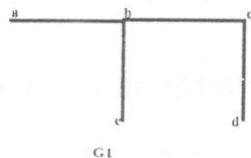
- c) Prove that  $(Z^+ / \sim)$  is partial order relation.

- d) Apply Prim's and Kruskal's algorithm on following digraph

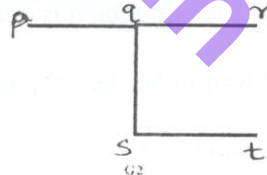


- e) If  $R$  is relation on  $Z$  defined  $(x, y) \in R$  iff  $3x + 5y$  is divisible by 8 show that  $R$  is an equivalence.

- f) Check the following graphs are isomorphic or not. Define Hand Shaking Theorem.



G1



G2

**Q.5) Solving any three of the following.**

(15)

- a) Calculate the expected value of  $X$  sum of the scores when two dice are rolled.

- b) A factory has two machines. The empirical evidence has established that machines I and II produce 30% and 70% output respectively. It has also been established that 5% and 1% produced by these machines respectively was defective. A defective item drawn at random. What is the probability that defective item was produced by machine I and II?

c) Use Pascal's formula to prove by mathematical induction that if  $n$  is an integer and  $n \geq 1$  then

$$\sum_{i=2}^{n+1} \binom{i}{2} = \binom{2}{2} + \binom{3}{2} + \dots + \binom{n+1}{2} = \binom{n+2}{3}$$

d) Consider two events  $A$  and  $B$  such that  $p(A) = 1/8$ ,  $p(A/B) = 1/4$ ,  $p(B/A) = 1/6$ . Examine the following statement and comments on validity of each of these i)  $A$  and  $B$  are independent ii)  $A$  and  $B$  are mutually exclusive iii) Occurrence of  $A$  implies that of  $B$  iv)  $p(A^c/B^c) = 0.5$

e) State Pigeon-Hole Principle. Show that if seven numbers from 1 to 12 are selected then at least two of them will add thirteen.

f) Consider four vowels and eight consonants i) find the number of five letter words containing two different vowels and three different consonants. ii) Find the number of five letter words if the words must contain the letter  $B$ . iii) Find the number of five letter words if the words must begin with  $B$  iv) Find the number of five letter words if they begin with  $A$  and contain  $B$ .

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