

Note – All Questions are compulsory.

Right indicates full marks

Q.1) Solve the following (any two)

(10)

a) Find A^{-1} by using Inversion Method If $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$

b) Check given systems is consistent/inconsistent if it is consistent then find the solution of given

system if $-X_1 - X_2 + 2X_3 + X_4 = 2$, $3X_1 + X_2 + X_3 + X_4 = 1$, $X_1 + 2X_2 + 2X_3 + 2X_4 = 3$.

c) Find the Rank of Matrix by Normal form : 1) $A = \begin{bmatrix} 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 \\ 4 & 5 & 6 & 7 \\ 9 & 10 & 11 & 12 \end{bmatrix}$ 2) $B = \begin{bmatrix} 1 & -1 & 3 & 6 \\ 1 & 3 & -3 & -4 \\ 5 & 3 & 3 & 11 \end{bmatrix}$

d) Determine the linearly dependent or Independent of vectors $(2, -1, 3, 2)$ $(1, 3, 4, 2)$ $(3, -5, 2, 2)$.

Q.2) Solve the following (any two)

(10)

a) Find the eigen values & eigen vectors corresponding to the following matrix $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$

b) Prove that given matrix is orthogonal & hence find A^{-1} If $A = \begin{bmatrix} \sqrt{2}/\sqrt{6} & 1/\sqrt{6} & \sqrt{3}/\sqrt{6} \\ \sqrt{2}/\sqrt{6} & -2/\sqrt{6} & 0 \\ \sqrt{2}/\sqrt{6} & 1/\sqrt{6} & -\sqrt{3}/\sqrt{6} \end{bmatrix}$

c) Express the Skew- Hermitian as $P + iQ$ if $A = \begin{bmatrix} 2+3i & 2 & 3i \\ -2i & 0 & 1+2i \\ 4 & 2+5i & -i \end{bmatrix}$

d) Check given matrix is Derogatory or Non – Derogatory $A = \begin{bmatrix} 7 & 4 & -1 \\ 4 & 7 & -1 \\ -4 & -4 & 4 \end{bmatrix}$

Q.3) Solve the following (any two)

(10)

a) Solve $(y^4 + 2y)dx + (xy^3 + 2y^4 - 4x)dy = 0$.

b) solve $\frac{dy}{dx} = \frac{x-y-1}{4y+x-1}$.

c) Solve $\frac{dy}{dx} = \frac{2y + \sqrt{x^2 + 4y^2}}{2x}$ by using $y = vx$.

d) Check exact differential equation if $\left[\log(x^2 + y^2) + \frac{2x^2}{x^2 + y^2} \right] dx + \frac{2xy}{x^2 + y^2} dy = 0$.

Q.4) Solve the following (any two)

(10)

a) State & prove Bernoulli's Theorem.

b) Solve Linear differential equation $\log y \frac{dx}{dy} + x = \log y$.

c) Solve $(D^3 - 5D^2 + 8D - 4)y = e^{2x} + 2e^x + 3e^{-x} + 2$.

d) Solve $x \frac{dy}{dx} + y = y^4 \log x$.

Q.5) Solve the following (any two)

(10)

a) State & prove Rolle's theorem & verify Rolle's theorem, if

$$f(x) = 2x^3 + x^2 - 4x - 2 \in [-\sqrt{2}, \sqrt{2}].$$

b) If $u = \tan^{-1} \frac{x^3 + y^3}{x - y}$ then show that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \sin 2u$.

c) If $u = \frac{1}{\sqrt{x^2 + y^2 + z^2}}$ then show that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$.

d) If $y = \log(cx + d)$ then Prove that $y_n = \frac{(-1)^{n-1}(n-1)!e^n}{(cx+d)^{n+1}}$.

(10)

Q.6) Solve the following (any two)

a) Find angle between the normal to the surface $xy = z^2$ at point $p(1, 1, 1)$ and $Q(4, 1, 2)$

b) Find the rate of change of $\phi = xyz$ in the direction normal to surface $x^2y + y^2x + z^2y = 3$ at point $(2, 2, 2)$

c) Prove that $\vec{a} = yz\vec{i} + xz\vec{j} + xy\vec{k}$ is irrotational and find the scalar function $\phi(xyz)$ is $\vec{a} = \text{grad of } \phi$.

d) Find directional derivative of the function $\phi = \left(\frac{1}{x^3 + y^3 + z^3}\right)^{1/3}$ at point $(2, -1, 2)$ in the direction $\vec{a} = \vec{i} + \vec{j} + \vec{k}$

(15)

Q.7) Solve the following (any three)

a) Find A^{-1} by adjoint if $A = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & -1 \\ 2 & 2 & 3 \end{bmatrix}$

b) If $\vec{a}$ is constant vector then show that $\vec{a} \times (\nabla \times \vec{r}) = \nabla(\vec{a} \cdot \vec{r}) - (\vec{a} \cdot \nabla)\vec{r}$.

c) Find maxima and minima at following if $f(x, y) = x^2 + y^2 + xy + x - 4y - 5$.

d) Solve $(D^2 - 2D + 5)y = 25x^2 + 12$.

e) Solve non exact differential equation $y(1 + xy + x^2y^2)dx + x(1 - xy + x^2y^2)dy = 0$

f) Show that $A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$ find i) A^T ii) A^{-1} iii) A^θ iv) $A^2 - 2A + I$ v) $A^3 + 2I$