Note- All questions are compulsory

Right indicate full marks

Q.1) Solve the following (any two) (10)

a) Find the A^{-1} by inversion method if matrix $A = \begin{pmatrix} 1 & -3 & 2 \\ 2 & 0 & 0 \\ 1 & 4 & 1 \end{pmatrix}$

b) Find the rank of matrix $A = \begin{pmatrix} 1 & 0 & 2 & -2 \\ 2 & -1 & 0 & -1 \\ 1 & 0 & 2 & -1 \\ 4 & -1 & 3 & -1 \end{pmatrix}$ and rank of $B = \begin{pmatrix} 6 & 1 & 3 & 8 \\ 4 & 2 & 6 & -1 \\ 10 & 3 & 9 & 7 \\ 16 & 4 & 12 & 15 \end{pmatrix}$

c) Check given system is consistent or in-consistent and find the values of unknowns

$x_1 + x_2 - x_3 + x_4 = 0$, $x_1 - x_2 + 2x_3 - x_4 = 0$, $3x_1 + x_2 + x_4 = 0$

d) Determine the value of p for which the following matrix A will have

(i) rank 1 , (ii) rank 2 , (iii) rank 3.

$$A = \begin{pmatrix} p & p & 2 \\ 2 & p & p \\ p & 2 & p \end{pmatrix}$$

Q.2) Solve the following (any two) (10)

a) Find the eigen value and eigen vector of following matrix

$$A = \begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$$

b) If $A = \begin{pmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{pmatrix}$ find (i) A^T (ii) $A^2 - 2A + I$ (iii) adj A. (iv) A^3

c) Prove that the given matrix is orthogonal and also find A^{-1}

If $A = \dfrac{1}{11} \begin{pmatrix} 2 & 6 & -9 \\ 6 & 7 & 6 \\ 9 & -6 & -2 \end{pmatrix}$

d) Show that given matrix are derogatory and find it's minimal polynomial

If matrix is $\begin{pmatrix} 7 & 4 & -1 \\ 4 & 7 & -1 \\ -4 & -4 & 4 \end{pmatrix}$

(P TO)

Q.3) Solve the following (any two)

a) Solve $\dfrac{dy}{dx} = \dfrac{2x-5y+3}{2x+4y-6}$ (10)

b) Solve $y/x \cos y/x \, dx - (x/y \sin y/x + \cos y/x) \, dy = 0$

c) Check it is exact or non-exact , if it is non-exact apply formula &find general solution if

$(2x\log x-xy)dy +y \, dx =0$

d) Determine orthogonal trajectories of all parabolas having vertices at (0,0) &foci on x-axis.

Q. 4) Solve the following (any two)

(10)

a) State &prove Bernoulli's differential equation

b) Solve $x\dfrac{dy}{dx} +y\log y =xye^{x}$.

c) Solve the linear differential equation $x\cos x \dfrac{dy}{dx} +y(x\sin x +\cos x) =1$.

d) Solve $(D^{3}-2D^{2}-5D+6)y =0, y(0) =0, y'(0) =0, y''(0) =1$

Q.5) Solve the following (any two)

(10)

a) If $A= 2yz\bar{i}-x^{2}y\bar{j}+x \, z^{2}\bar{k}$, $B=x^{2}\bar{i} + yz\bar{j} - xy\bar{k}$ and $\emptyset=2x^{2}yz^{3}$, find

(i) $A\cdot\nabla\emptyset$, (ii) $(A \times \nabla) \emptyset$,(iii) $A \times \nabla\emptyset$

b) Prove that $A=(z^{2}+2xy+3y)\bar{i}+(3x+2y+z)j+(y+2zx)k$ is irrotational and find scalar potential $\emptyset$ such that

$A =\nabla\emptyset$ and $\emptyset (1,1,0)=4$.

c) Find the acute angle between the surfaces $x^{2} + y^{2} + z^{2}=9$ and $z=x^{2}+y^{2}-z$ at the point (2.-1,2).

d) Prove that the four points whose position vectors are $3i-2j+4k, 6i +3j+k, 5i+7j+3k, 2i+2j+6k$ are coplaner.

Q.6) Solve the following (any two)

(10)

a) If $u=\log (x^{2}+y^{2}+z^{2})$, prove that $z\dfrac{\partial 2 u}{\partial x\partial y} = y \dfrac{\partial 2u}{\partial z\partial x} = x\dfrac{\partial 2u}{\partial y\partial z}$.

b) State &prove Langranges Mean Valued Theorem.

c) If $u = \dfrac{x2+y2+z2}{x+y+z}$ Verify Euler Theroem .

d) Find the nth order derivative of $y = x^{2}/2x^{2} +7x+6$.

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