

- Right indicates full marks
- All questions are compulsory

Q.1. Solve the following. (any two) (10)

a) Find  $A^{-1}$  by Inversion method

$$\text{If } A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 2 & -3 \\ 1 & -4 & 9 \end{bmatrix}$$

b) Check following vectors are linearly Dependent or Independent if

$$[1 \ 2 \ -1 \ 0] \ [1 \ 3 \ 1 \ 2] \ [4 \ 2 \ 1 \ 0] \ [6 \ 1 \ 0 \ 1]$$

c) Test for consistency the following equation and if possible solve them.

$$x_1 - x_2 + 2x_3 + x_4 = 2; \ 3x_1 + 2x_2 + x_4 = 1; \ 4x_1 + x_2 + 2x_3 + 2x_4 = 3$$

d) Find Rank of following matrix (i) by normal form and (ii) by echelon form

$$\text{i) } \begin{bmatrix} 2 & 1 & 4 & -1 \\ 1 & 2 & 1 & 3 \\ 4 & 5 & -1 & 2 \\ 8 & 7 & 7 & 3 \end{bmatrix} \quad \text{ii) } \begin{bmatrix} 2 & -1 & 3 & 4 \\ 0 & 3 & 4 & 1 \\ 2 & 3 & 7 & 4 \\ 2 & 5 & 11 & 6 \end{bmatrix}$$

Q.2. Solve the following. (any two) (10)

a) Find Eigen value and Eigen vector of following matrix

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 2 & 3 & 2 \\ 3 & 3 & 4 \end{bmatrix}$$

b) Verify Cayley's theorem

$$A = \begin{bmatrix} 7 & -1 & 3 \\ 6 & 1 & 4 \\ 2 & 4 & 8 \end{bmatrix}$$

c) If  $A = \begin{bmatrix} 1/3 & 2/3 & a \\ 2/3 & 1/3 & b \\ 2/3 & -2/3 & c \end{bmatrix}$  is orthogonal and also find  $A^{-1}$ .

d) Express in  $p + iQ$  where  $p$  is a real symmetric and  $Q$  is a real skew symmetric if

$$A = \begin{bmatrix} 3i & -1+i & 3-2i \\ 1+i & -i & 1+2i \\ -3-2i & -1+2i & 0 \end{bmatrix}$$

Q.3. Solve the following. (any two)

(10)

- $y^2 + x^2 \frac{dy}{dx} = xy \frac{dy}{dx}$
- $(2x + 3y - 1)dx + (3x - 2y + 1)dy = 0$
- $(1 + x^2) \frac{dy}{dx} + y = e^{\tan^{-1}x}$
- State and prove Bernoulli's theorem.

Q.4. Solve the following. (any two)

(10)

- Find  $n^{th}$  derivative of  $\frac{x}{(x-1)(x-2)(x-3)}$ .
- If  $u = e^{xyz}$  then prove that  $\frac{\partial^2 u}{\partial x \partial y \partial z} = (1 + 3xyz + x^2 y^2 z^2) e^{xyz}$ .
- State and prove Rolle's theorem and verify Rolle's theorem if  $f(x) = x^2 - 4$  in  $[0, 4]$ .
- Find maximum and minimum value of  $f(x, y)$  if  $f(x, y) = y^2 + 4xy + 3x^2 + x^3$

Q.5. Solve the following. (any two)

(10)

- Calculate  $\nabla^2 f$  when  $f = 3x^2z - y^2z^3 + 4x^3y + 2x - 3y - 5$  at point  $(1, 1, 0)$ .
- Prove that  $\vec{A} = (6xy + z^3)\vec{i} + (3x^2 - z)\vec{j} + (3xz^2 - y)\vec{k}$  is irrotational and find  $\phi$  such that  $\vec{A} = \nabla\phi$ .
- Find angle between the normal to the surfaces  $xy = z^2$  at  $P(1, 1, 1)$  &  $Q(4, 1, 2)$ .
- Find Curl ( $\text{curl } \vec{A}$ ) =  $x^3y\vec{i} - 2x^2z\vec{j} + 3yz\vec{k}$  at  $(1, 3, 4)$ .

Q.6. Solve the following. (any two)

(10)

- Solve  $(D^3 - 2D^2 - 5D + 6)y = 0$   $y(0) = 0, y'(0) = 0, y''(0) = 1$ .
- Solve  $(D^6 - 64)y = e^x \cos h 2x$ .
- Solve  $(D^2 + 3D + 2)y = \sin 2x$ .

d) Solve  $(D^2 - 2D + 5)y = 25x^2 + 12$ .

7. Solve the following. (any three)

(15)

a) Verify Euler's theorem  $u = \frac{x+y+z}{\sqrt{x}+\sqrt{y}+\sqrt{z}}$

b) Find  $A^{-1}$  by adjoint method if  $A = \begin{bmatrix} 1 & 5 & 3 \\ 2 & 1 & 3 \\ 2 & 5 & 2 \end{bmatrix}$ .

c) If  $\vec{A} = 2xyz\vec{i} - x^2y\vec{j} + xz^2\vec{k}$ ,  $\vec{B} = x^2\vec{i} + yz\vec{j} - xy\vec{k}$  &  $\phi = 2x^2yz^3$  then find  
i)  $(\vec{A}, \nabla)\phi$  ii)  $(\vec{B}, \nabla)A$  iii)  $\vec{A} \cdot \nabla\phi$

d) Solve  $xe^x(dx - dy) + e^x dx + ye^y dy = 0$  check exact if not then apply particular formula and find general solution.

e) Check the given matrix is Derogatory or non derogatory

$$\begin{bmatrix} 7 & 4 & -1 \\ 4 & 7 & -1 \\ -4 & -4 & 4 \end{bmatrix}$$

f) Solve i)  $(D^4 - 6D^3 + 12D^2 - 8D)y = 0$

ii)  $2D^3y - 2Dy - y = 0$