07/10/14

VCD – F.Y.I.T. APPLIED MATHS I – SEM I – 75 MARKS – 2 ½ HRS

Right indicates full marks

All questions are compulsory

Solve the following. (any two) (10)

a) Find A^{-1} by Inversion method

$$\text{If } A = \begin{bmatrix} 1 & -3 & 2 \\ 2 & 0 & 0 \\ 1 & 4 & 1 \end{bmatrix}$$

b) Check following vectors are linearly Dependent or Independent if

$X_1 = [1 \quad 0 \quad 2 \quad 1]\ [3 \quad 1 \quad 2 \quad 1]\ [4 \quad 6 \quad 2 \quad -4]\ [-6 \quad 0 \quad -3 \quad -4]$

c) Test for consistency the following equation and if possible solve them.

$x_1 - x_2 + x_3 - x_4 = 1$; $2x_1 - x_2 + 3x_3 = 2$; $3x_1 - 2x_2 + 2x_3 + x_4 = 1$; $x_1 + x_3 + 2x_4 = 0$

d) Find Rank of following matrix (i) by normal form and matrix (ii) by echelon form

i) $\begin{bmatrix} 1 & 2 & 3 & -1 \\ -2 & -1 & -3 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$ ii) $\begin{bmatrix} 1 & 3 & 2 & 2 \\ 1 & 2 & 1 & 3 \\ 2 & 4 & 3 & 4 \\ 3 & 7 & 4 & 8 \end{bmatrix}$

Solve the following. (any two) (10)

a) Find Eigen value and Eigen vector of following matrix

$$A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$$

b) Verify Caley's theorem

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$$

c) If $A = \begin{bmatrix} 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ \sqrt{2}/3 & 1/\sqrt{6} & 1/\sqrt{6} \\ 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} \end{bmatrix}$ is orthogonal and also find A^{-1}.

d) Express in $p + iQ$ where p is a real symmetric and Q is a real skew symmetric if

$$A = \begin{bmatrix} i & 1-i & 2+3i \\ -1-i & 2i & -3i \\ -2+3i & -3i & -i \end{bmatrix}$$

Q.3. Solve the following. (any two) (10)

a) $\left(4 - \frac{y^2}{x^2}\right) dx + \frac{2y}{x} dy = 0$

b) $(2x + 4y + 3)\frac{dy}{dx} = (2y + x + 1)$

c) $\frac{dy}{dx} + (\cot x)y = \cos x$

d) State and prove Bernoulli's theorem.

Q.4. Solve the following. (any two) (10)

a) Find n^{th} derivative of $\frac{2}{(x-1)(x-2)(x-3)}$.

b) If $u = \log(x^2 + y^2 + z^2)$ then prove that $x\frac{\partial^2 u}{\partial y\,\partial z} = z\frac{\partial^2 u}{\partial x\,\partial y} = y\frac{\partial^2 u}{\partial z\,\partial x}$.

c) State and prove Cauchy's Mean Value Theorem

d) Find maximum and minimum value of $f(x, y)$ if $f(x, y) = x^3y^2(1 - x - y)$.

Q.5. Solve the following. (any two) (10)

a) Prove that $\nabla \times (\bar{a} \times \bar{r}) = 2\bar{a}$ where a is a constant vector.

b) Prove that $\bar{A} = (z^2 + 2x + 3y)\bar{i} + (3x + 2y + z)\bar{j} + (y + 2zx)\bar{k}$ is conservative and find scalar potential φ such that $\bar{A} = \nabla\varphi$.

c) Find angle between the normal to the surfaces

$ax^2 + y^2 + z^2 - 1$ & $bx^2y + y^2z + z = 1$ at $(1,1,0)$.

d) Find Curl $(curl\ \bar{A}) = 2yz\bar{i} - x^2y\bar{j} + xz^2\bar{k}$ at $(1, -3, 2)$.

6. Solve the following. (any two) (10)

a) Solve $(4D^3 - 4D^2 - 9D + 9)y = 0 \quad y(0) = 1, y'(0) = 0, y''(0) = 1.$

b) Solve $(D^2 - 9)y = x + e^{2x} - \sin 2x.$

c) Solve $(D^2 - 5D + 6)y = x \cos 2x.$

d) Find the equation of the family of all trajectories of the family of circles which pass through origin (0,0) & have center on Y- axis.

7. Solve the following. (any three) (15)

a) Verify Euler's theorem $u = x^2yz - 4y^2z^2 + 2xz^3$.

b) Find A^{-1} by adjoint method if $A = \begin{bmatrix} 1 & 3 & 4 \\ 4 & 2 & 1 \\ 1 & 1 & 3 \end{bmatrix}$.

c) If $\bar{A} = x^2\bar{\imath} - xye^x\bar{\jmath} + \sin\bar{k}$ then find $\nabla.(\nabla \times \bar{A})$

d) Solve $(2xy^4e^y + 2xy^3 + y)dx + (x^2y^4e^y - x^2y^2 - 3x)dy = 0$

e) Check the given matrix is Derogatory or non derogatory

$$\begin{bmatrix} 2 & -1 & 1 \\ 2 & 2 & -1 \\ 1 & 2 & -1 \end{bmatrix}$$

f) If $u = x^3y + e^{xy^4}$ then prove that $\frac{\partial^2 u}{\partial x \, \partial y} = \frac{\partial^2 u}{\partial y \, \partial x}$