28/10/23
VCD SYDS-SEM-III-Linear Algebra and Discrete Mathematics-$2\frac{1}{2}$hrs-MARKS-75

Note: (i) All questions are compulsory.
(ii) Figures to the right indicate marks.
(iii) Illustrations, in-depth answers and diagrams will be appreciated.
(iv) Mixing of sub-questions is not allowed.

Q.1) Attempt the following (Any Three) **(15M)**

a) Define subspace of a vector space V.
Verify the set W= $\{(x,0)/x \in IR\}$ is subspace of IR^2.

b) Solve the non-homogenous linear equations
$x + y + z = 1, x + 2y + 3z = 5, 2x + 3y + 4z =$

c) Prove all five addition properties of vector space for the set $S = \{(3x, x): x, y \in \mathbb{R}\}$

d) Define linearly dependent and independent vectors.
Verify S = {(-1,1,1),(2,1,1),(1,2,2)} is linearly dependent or independent.

e) Define linear combination of vectors in vector space V. Verify (1,2,3) as a linear combination of (1,0,0), (1, 1,0) and (1,1,3).

f) Let $T: \mathbb{R}^2 \to \mathbb{R}^2$ defined by $T(x,y) = (x - y, 1)$. Show that T is not a linear Transformation.

Q.2) Attempt the following (Any Three) **(15M)**

a) Find Area of tetrahedron formed by (1,1,3),(4,3,2),(5,2,7),(6,4,8)

b) Find distance and angle between two vectors x= (-2,1,3) and y=(4,5,1) in IR^3

c) Use the Gram-Schmidt orthonormalization process to construct an orthonormal basis for the set S={(2,1,2),(4,1,0),(3,1,-1)}

d) Check whether the set $\{(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0), (\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0), (0,0,1)\}$ is orthonormal.

e) Using Crammer's rule, Solve the non-homogenous linear equations
$x + y - z = 1,\ 8x + 3y - 6z = 1,\ -4x - y + 3z = 1$

f) Find i) volume of Parallelopiped formed by (3,2,1),(-1,3,0), (2,2,5).
ii) area of a triangle formed by (-4,1) ,(6,2), (3,-3)

Q.3) Attempt the following (Any Three) **(15M)**

a) Show that Similar matrices have same eigen values.

b) Solve the following differential equation using Eigen values and Eigen vectors

$$\frac{dx}{dt} = 2x + 3y, \qquad \frac{dy}{dt} = 4x + y$$

c) Prove that eigen value of A is same eigen value of A^T.

d) Check whether the matrix A= $\begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$ is diagonalizable. If yes, then non-singular matrix P such that $P^{-1}AP$ is diagonal matrix.

e) Check whether the matrix $A = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$ is orthogonal or not.

f) Find Eigen values and Eigen vectors for the matrix $A=\begin{bmatrix}-2 & 0\\ 0 & 4\end{bmatrix}$

Q.4) Attempt the following (Any Three) **(15M)**

a) Find singular value decomposition of $A = \begin{bmatrix}1 & 1 & 0\\ 0 & 1 & 1\end{bmatrix}$

b) Define norm of a matrix. Find the condition number of the positive definite matrix $A = \begin{bmatrix}3 & 4\\ 4 & 6\end{bmatrix}$.

c) Find minimum value of the given quadratic form:
$P(x, y) = 4x^2 - 2xy + 3y^2 + 3x\text{-}2y$

d) Find local maxima, local minima and saddle point of the function
$f(x,y) = x^3 + 3xy^2 - 15y^2 - 15x^2 + 72x$.

e) Check whether the following matrices are positive definite or not.
$A = \begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}$ and $B=\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}$

f) Explain the types of Quadratic forms.

Q.5) Attempt the following (Any Three) **(15M)**

a) Find Optimal Solution by simplex method

Maximize $z = 3x_1 + 7x_2$

Subject to constrains $2x_1 + 5x_2 \leq 20$

$x_1 + 2x_2 \leq 4$

$x_1 \geq 0, x_2 \geq 0$

b) Solve the given linear programming problem graphically.

Maximize $z = 13x + 15y$

Subject to $5x + 8y \leq 760$

$3x + y \leq 228$

$x \geq 0, y \geq 0$

c) Following payoff matrix refers to two player game player A and player B. Each player has four strategic options. Find optimal strategies for player A and B in the following game. Also find the value of the game.

	Player B			
Player A	500	260	200	210
	−50	−100	−40	240
	200	400	100	−20
	250	300	100	50

d) Explain Duality in L.P.P. Also Write the dual of the given problem:

Minimize $z = 4x_1 + 3x_2$

Subject to $200x_1 + 100x_2 \geq 4000$

$x_1 + 2x_2 \geq 50$

$40x_1 + 40x_2 \geq 1400$

$x_1 \geq 0, x_2 \geq 0$

e) Solve the given linear programming problem graphically.

Minimize $z = 10x + 7y$

Subject to $2x + y \geq 2$

$x + 3y \geq 3$

$x \geq 0, y \geq 0$

f) Two types of food i.e. A and B are available. Each contains vitamin N1 and N2. A person needs 4 gram of vitamin N1 and 12 grams vitamin N2 per day. Food packet A contain 2 gram of vitamin N1 and 4 grams of vitamin N2. Food packet B contain 1 gram of vitamin N1 and 4 grams of vitamin N2 . If the food packet cost Rs.15 and Rs.10 for A and B respectively, Then Formulate the LPP to minimize the cost.