VCD: 111022 S.Y DS/SEM-III/LA & DM/75 marks/2 hours 30 min

Note: All questions are compulsory

Q1. Attempt any three of the following: **(15 marks)**

a. Find inverse of the matrix A= $\begin{bmatrix} 2 & 3 & 4 \\ 4 & 3 & 1 \\ 1 & 2 & 4 \end{bmatrix}$

b. Solve the non-homogenous linear equations
$x + y + z = 3, x - 2y - 3z = 1, 2x + 3y + z = 9$

c. Define linear combination of vectors in vector space V. Verify $\begin{bmatrix} 7 & 5 \\ 2 & -1 \end{bmatrix}$ is linear combination of $\begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 5 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 2 & 1 \end{bmatrix}$.

d. Verify the set S= {(1,0), (0,1)} is basis of R^2.

e. Define Subspace of vector space V.
Verify the set W= $\{(x, x^2): x, y \in \mathbb{R}\}$ is subspace of R^2.

f. If A and B are two n X n non-singular matrices then prove that AB is also non-singular and prove that $(AB)^{-1} = B^{-1}A^{-1}$.

Q2. Attempt any three of the following: **(15 marks)**

a. Find projection of vector b = (1, 1, 2) onto a = (1, 2, 1). Also, find projection of vector b = (1, 1, 2) orthogonal to a = (1, 2, 1).

b. Use Gram Schmidt orthogonalization process to construct orthonormal basis of R^3 for S = {(1, 0, 1), (1, 0, -1), (0, 3, 4)}

c. Find distance and angle between the vectors (-2, 1, 3) and (4, 5, 1).

d. Check whether the set $\{\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right), \left(\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}, 0\right), (0,0,1)\}$ is orthonormal.

e. Using Cramer's rule, solve the following equations:
x + y - z = 1, 8x + 3y - 6z = 1, -4x - y + 3z = 1

f. i) Find the area of the triangle formed by (1, 2), (7, 1), (3, 5).
ii) find the volume of parallelopiped formed by (3, 2, 1), (-1, 3, 0), (2, 2, 5).

Q3. Attempt any three of the following: **(15 marks)**

a. Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 1 & 1 \\ -2 & 4 \end{bmatrix}$

b. Check whether the following matrix is diagonalizable. If yes, then find non singular matrix P such that $P^{-1}AP$ is a diagonal matrix.

$$A = \begin{bmatrix} 7 & 2 \\ -4 & 1 \end{bmatrix}$$

c. Solve the following differential equation using eigen values and eigen vectors

$$\frac{dx}{dt} = 6x + 5y$$

$$\frac{dy}{dt} = x + 2y$$

d. Solve the difference equation $u_{k+1} = Au_k$ where $A = \begin{bmatrix} 5 & -3 \\ -6 & 2 \end{bmatrix}$ and $u_0 = 3$.

e. Define Hermitian and skew Hermitian matrices. Prove that eigen value of every Hermitian matrix is real.

f. i) Define similar matrices. Show that similar matrices have same eigen values.
ii) If A and B are similar matrices and X is the eigen vector of A w.r.t eigen value μ, then what is the eigen vector of B w.r.t eigen value μ?

Q4. Attempt any three of the following: (15 marks)

a. Find local maxima, local minima and saddle points of the function
$f(x,y) = x^2 + y^2 + xy + 3x + 3y + 7$

b. Check whether the following matrices are positive definite or not.

$$A = \begin{bmatrix} 3 & 4 \\ 4 & 6 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

c. Find singular value decomposition of A = $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$.

d. Find minimum value of quadratic form $P(x,y) = 4x^2 - 2xy + 3y^2 + 3x - 2y$.

e. Find the condition number of the symmetric matrix $A = \begin{bmatrix} 3 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 2 & 3 \end{bmatrix}$ for spectral norm.

f. Define 5 different types of quadratic forms.

Q5. Attempt any three of the following: (15 marks)

a. A factory manufactures two products A and b. To manufacture one unit of A, 1.5 machine hours and 2.5 labour hours are required. To manufacture one unit of product B, 2.5 machine hours and 1.5 labour hours are required. In a month, 300 machine hours and 240 labour hours are available. Profit per unit, for A is ₹ 50 and for B is ₹ 40. Formulate the linear programming problem to maximize the profit.

b. Solve the given linear programming problem graphically:

Max z = 5x + 8y

subject to

$3x + 2y \leq 24$

$x + 3y \leq 12$

$x, y \geq 0$

c. Solve the given linear programming problem using simplex method:

Max z = 8x + 20y

subject to

$2x + y \leq 80$

$3x + 4y \leq 96$

$x, y \geq 0$

d. Write a short note on duality. Also, write the dual of the given problem:

Max z = $9x_1 + 4x_2$

subject to

$4x_1 + 3x_2 \leq 12$

$4x_1 + x_2 \leq 8$

$4x_1 + 9x_2 \leq 8$

$x_1, x_2 \geq 0$

e. Following payoff matrix refers to a two player game, player X and player Y. Each player has 3 strategy options.

i) Find the optimal strategy for both the players.

ii) Find the value of the game.

Player X

$$\text{Player Y}\begin{bmatrix} 220 & 150 & 170 \\ -50 & 40 & -20 \\ 140 & 120 & 100 \end{bmatrix}$$

f. Find the maximum flow of the given network (capacity of each edge is given):