VCD.110522.FYDS SEM II PROBABILITY AND DISTRIBUTION 75 MAKES 2 HRS 30 MIN

Q. 1) Attempt the following **[40 MARKS]**

1) A variable that can assume any possible value between two points is called
   a) Discrete random variable b) Continuous random variable
   c) Discrete sample space d) Random variable

2) The probability function of a random variable is defined as: X: -1 -2 0 1 2, f(x): k 2k 3k 4k 5k,
   Then k is equal to:
   a) Zero b) 1/4 c) 1/15 d) One

3) The area under the curve of a continuous probability density function is always =
   a) Zero b) One c) -1 d) None

4) The formula of variance of uniform distribution is
   a) $(b-a)^2/6$ b) $(b-a)^2/12$ c) $(b-a)^3/8$ d) $(b+a)^2/2$

5) The p.d.f. of normal distribution is
   a) f(x)= 1/(σ√2π) e^(-1/(2σ²) (x-μ)²) b) f(x)= 1/√2π e^(-1)
   c) f(x)= 1/(σ√2π) e^1 (x-μ)² d) f(x)= 1/(σ√2π)

6) __________ plays an important role in the theory of Reliability and Survival analysis and Queuing theory.
   a) Triangular distribution b) Uniform Continuous distribution
   c) Exponential distribution d) Gamma distribution

7) If X follows a Gamma distribution with parameter (α, λ = 1) then it is a probability density function of with parameter α
   a) Gamma distribution Beta b) type-2 distribution
   c) Uniform continuous distribution d) Exponential distribution

8) If random variable X follows Gamma (α =2 , λ =5) then the value of its Mean is _____
   a) 25/2 b) 5/4 c) 5/2 d) 4/5

9) Area to the right of mean under the normal curve is_____.
   a) 68.27% b) 50% c) 90% d) 95%

10) Set of all possible outcomes is known as _____.
   a) Probability b) Random experiment c) Sample space d) Event

11) Two events A and B are mutually exclusive events if _____.
   a) $A \cup B = S$ b) $A \cup B = \emptyset$ c) $A \cap B = S$ d) $A \cap B = \emptyset$

12) If A and B are mutually exclusive events then $P(A \cup B)$ = _____
   a) $P(A) + P(B) - P(A \cap B)$ b) P(A) + P(B) c) P(A)·P(B) d) Zero

13) P(S/B) = _____
   a) P(B) b) 1 c) 0 d) Not fixed

14) A variable which can assume finite or countably infinite number of values is known as_____.
   a) continuous b) discrete c) qualitative d) quantitative

15) For Probability mass function, following condition /conditions satisfied
   a) $P(x_i) \geq 0$ b) $\sum P(x_i) = 1$ c) $P(x_i) \geq 0$ and $\sum P(x_i) = 1$ d) $P(x_i) \geq 0$ or $\sum P(x_i) = 1$

16) Which of the following statement is true for cumulative distribution function F(x)?

a) F(x) is decreasing function b) F(x) ranges from 0 to ∞

c) F(x) is non-decreasing function d) $P(a < x \le b) = F(b) - F(a) - P(b)$

17) For Probability density function f(x), following condition /conditions satisfied

a) $f(x) \ge 0, \forall x \in (-\infty, \infty)$ b) $\int_{-\infty}^{\infty} f(x)dx = 1$

c) $f(x) \ge 0, \forall x \in (-\infty, \infty)$ and $\int_{-\infty}^{\infty} f(x)dx = 1$ d) $f(x) \ge 0, \forall x \in (-\infty, \infty)$ or $\int_{-\infty}^{\infty} f(x) = 1$

18) For discrete random variable, the expected value E(x) = ______.

a) $\sum x$ b) $\sum xP(x)$ c) $\sum P(x) + x$ d) $\sum xP^2(x)$

19) The characteristic function of random variable X is ______

a) $\emptyset(t) = e^{itx}$ b) $\emptyset(t) = E(e^{itx})$ c) $\emptyset(t) = e^{tx}$ d) $\emptyset(t) = E(e^{tx})$

20) E (aX+bY) = ______________.

a) E(X) + E(Y) b) aE(X) + bE(Y) c) E(X) E(Y) d) ab E(X) E(Y)

Q. 2 A) Attempt the following (Solve any 01) **[04 MARKS]**

1) Define the following terms with suitable example

a) Impossible event b)Simple event c)Independent event d)Mutually exclusive

2) Prove the following statement

a) P (A') = 1- P (A), where A' is complementary event of A.

b) If A and B are mutually exclusive events then P (A∪B) = P (A) + P (B)

B) Attempt the following (Solve any 01) **[03 MARKS]**

1) 5 Indians and 3 Americans stand for photograph randomly. Find the probability that

a) Two extreme positions are occupied by Indians. b) American are all adjacent.

2) For two events A and B , if P(A) = 0.98 and P(A∪B) = 0.9 ,then find P(B)

a) If A and B are independent . b) A and B are mutually exclusive

Q. 3 A) Attempt the following (Solve any 01) **[04 MARKS]**

1) A random variable X has the following probability distribution with p.m.f. p(X)

X	0	1	2	3	4	5	6	7
P(X)	0	$2k$	$3k$	k	$2k$	k^2	$7k^2$	$2k^2 + k$

a) Find the value of k

b) Find P(X < 3)

2) Verify the following function is probability density function

$$f(X) = \begin{cases} \frac{X^3(10-X)}{5000}, & for\ 0 \le X \le 10 \\ = 0 , & otherwise \end{cases}$$

B) Attempt the following (Solve any 01) **[03 MARKS]**

1) Define the following.

a) Random variable. b) Discrete random variable with suitable example.

c) Continuous random variable with suitable example.

2) Define Cumulative distribution function for continuous random variable.

Also state the properties of Cumulative distribution function.

Q. 4 A) Attempt the following (Solve any 01) **[04 MARKS]**

1) Define Mathematical Expectation of discrete random variable (univariate).Also prove that
   a) $E(aX+b) = a\,E(X) + b$ b) $E(b) = b$

2) A random variable X has probability mass function as,

$$P(X = x) = \begin{cases} \frac{x}{15}, & X = 1,2,3,4,5 \\ = 0, & otherwise \end{cases}$$

   Then obtain Moment generating function. Also find mean.

B) Attempt the following (Solve any 01) **[03 MARKS]**

1) If the bivariate probability mass function is given by

$$P_{X,Y}(x, y) = \begin{cases} k(2x + y), & x, y = 2,3,4 \\ = 0, & otherwise \end{cases}$$

   Then find a) value of k b) marginal pmf of X

2) Define Characteristic function. State all its properties.

Q. 5 A) Attempt the following (Solve any 01) **[04 MARKS]**

1) Derive mean of binomial distribution. Also find recurrence relations of probabilities of binomial distribution.

2) Define hypergeometric distribution. Also find it's mean.

B) Attempt the following (Solve any 01) **[03 MARKS]**

1) For Poisson variate X, $P(X=1) = P(X=2)$. Find $P(X=4)$

2) In laboratory there are 6 non-defective computers and 4 defective computers. A random sample of 5 computers is selected state the probability distribution of X, the number of defectives in sample. Find the probability that the sample contains at least one defective computer.

Q. 6 A) Attempt the following (Solve any 01) **[04 MARKS]**

1) Define rectangular distribution. Also write it's cumulative distribution function. And draw it's graph of pdf and cdf.

2) Define normal distribution. Also write its properties briefly.

B) Attempt the following (Solve any 01) **[03 MARKS]**

1) Define standard normal distribution. If $Z \sim N(0, 1)$ find $P(Z<1)$ for $P(Z=1) = 0.3413$.

2) The waiting time are a bus stop is assured to follow rectangular distribution over (5, 15). What is the chance that a person arriving at bus stop gets bus Between 8 to 12 min.