

1. All questions are compulsory.
2. All questions carry equal marks.
3. Draw neat, labeled diagrams wherever necessary.

1. Attempt the following (Any three) (15 M)

- a. Find  $f \circ g, g \circ f, f \circ f$  and  $g \circ g$  of the functions  $f(x) = x^2, g(x) = x + 1$
- b. Find the inverse of the function  $f(x) = \frac{3-4x}{8x-1}$
- c. A quadratic function  $f(x) = 2x^2 + 12x + 10$  is given. (i) Express  $f$  in standard form. (ii) Sketch a graph of  $f$ . (iii) Find the maximum or minimum value of  $f$ .
- d. Express the polynomial  $f(x) = x^4 - 3x^3 + 2x^2$  into factor form and also find all its zeroes
- e. Find an equation of the circle that has the points  $P(1, 8)$  and  $Q(5, -6)$  as the endpoints of a diameter
- f. Solve :  $(x - 4)(x + 2)^2 < 0$

2. Attempt the following (Any three) (15 M)

- a. Solve :  $\log_3(x + 15) - \log_3(x - 1) = 2$
- b. A certain culture of the bacterium *Rhodo-bactersphaeroides* initially has 25 bacteria and is observed to double every 5 hours. (a) Find an exponential model for the number of bacteria in the culture after  $t$  hours. (b) Estimate the number of bacteria after 18 hours. (c) After how many hours will the bacteria count reach 1 million?
- c. Find the exact value of the following:  
i)  $\cos \frac{19\pi}{6}$       ii)  $\tan \frac{5\pi}{3}$
- d. Find the values of all the trigonometric functions of  $t$  from  $\tan t = \frac{1}{4}$ , terminal point of  $t$  is in Quadrant III
- e. Find the value of the following: i)  $\tan^{-1}(-\sqrt{3})$     ii)  $\cos(\tan^{-1} \frac{4}{3})$
- f. Find the area of the triangle having sides of length 7 and 9 and included angle  $72^\circ$

3. Attempt the following (Any three) (15 M)

- a. Verify the identity:  $\frac{1+\tan^2 x}{1-\tan^2 x} = \frac{1}{\cos^2 x - \sin^2 x}$
- b. Prove the identity:  $\frac{1-\sin x}{1+\sin x} = (\sec x - \tan x)^2$
- c. Find  $\tan 2\theta$  if  $\cos \theta = \frac{3}{5}$  where  $\theta$  is in quadrant I.
- d. Find the value of the following: i)  $\cos 75^\circ$     ii)  $\tan \frac{\pi}{12}$
- e. Solve the equation:  $2\sin^2 \theta - 7\sin \theta + 3 = 0$
- f. Find all the solution of  $2\sin 3\theta - 1 = 0$

4. Attempt the following (Any three) (15 M)

- a. Express complex number  $2\sqrt{3} - 2i$  into polar form
- b. Find the fifth root of  $Z = 1 + i$

- c. Solve the following system of equation by using Gaussian Elimination method

$$\begin{cases} x - 2y + z = 1 \\ y + 2z = 5 \\ x + y + 3z = 8 \end{cases}$$

- d. Solve the following system of equation by using Cramer's rule

$$\begin{cases} 2x - 5y = 4 \\ x + y - z = 8 \\ 3x + 5z = 0 \end{cases}$$

- e. If  $u = 2i + j, v = 3i - 2j$  find  $u \cdot v$  and the angle between the vectors  $u$  and  $v$   
f. If  $u = -j + 3k, v = 2i - k$ , find a unit vector that is orthogonal to the plane containing the vectors  $u$  and  $v$ .

5. Attempt the following (Any three)

(15 M)

- a. Find the equation of the parabola that has its vertex at the origin and directrix  $x = -5$ . And sketch its graph.  
b. Find the vertices, foci, asymptotes, length of transverse axis of the hyperbola  $25y^2 - 9x^2 = 225$  and sketch its graph.  
c. The 3rd term of a geometric sequence is  $63/4$  and the 6th term is  $1701/32$ . Find the fifth term.  
d. Use Mathematical induction prove that  $1^2 + 2^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$  for all natural number 'n'  
e. Find the limit if it exist: i)  $\lim_{x \rightarrow -2} \frac{4x^3 + 2x^2 - 1}{5 - 3x}$  ii)  $\lim_{x \rightarrow 0} \frac{|x|}{x}$   
f. Find the derivative of  $f(x) = \sqrt{x}$  at  $x = a$

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